F2-Bubbles: Faithful Bubble Set Construction and Flexible Editing

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Abstract—In this supplemental material, we document contents that are left out from the paper for conciseness. We start by showing the analysis of computational complexity of our method. Then, full results of the evaluation are reported. Finally, all visualizations of all datasets used in the evaluation are shown.

1 Time Complexity Analysis of the Joint Construction of Spanning Trees Algorithm

In this section, we provide the detailed time complexity analysis of the joint spanning tree construction algorithm. To facilitate the explanation, we include it as Algorithm 1.

Algorithm 1 Joint Construction of Spanning Trees

Require: an array of points $\mathbf{P}$ with k sets
Ensure: a forest F with k spanning trees
1: initialize a forest $F = \{\mathbb{E}, \mathbb{V}\}$, where $\mathbb{E} = \emptyset$ and $V = \mathbf{P}$
2: construct a graph G with k complete sub-graphs based on $\mathbf{P}$
3: calculate edge weights in G with Eq.3 of the paper
4: find the edge e_{ob} with the minimum weight in G
5: **repeat**
6: update the weights of un-selected edges in G with e_{ob}
7: add the edge e_{nb} with the minimum weight and set $e_{ob} = e_{nb}$
8: run surface routing algorithm for the new edge
9: **until** F has only k trees

The time complexity of Algorithm 1 is analyzed line by line. For a dataset of n nodes with k sets, we assume that some nodes belongs to multiple sets. To derive the bound of the complexity, we analyze two cases: i) the data set without any node duplication, and ii) all the nodes duplicated for k times to form k sets.

We start by analyzing the non-duplicate case. The time complexities for initializing the forest F (Line 1) and constructing the complete graph G (Line 2) are $O(n)$, and $O(m)$, respectively, in which m is the number of the edges in G. Edge weight computation (Line 3) requires intersection tests for all pairs of nodes in G, which yields a complexity of $O(m^2)$. Finding the minimal weight (Line 4) takes $O(m \log m)$ to initialize the heap, and $O(\log m)$ to find the minimal. The analysis of the loop (Lines 5–9) is as follows. The update of the weight of un-selected edges in G (Line 6) needs to compute the intersection of e_{ob} against all other edges, and is of $O(m)$. Line 7 has a total complexity of $m(O(\log m) + O(\log n)) = O(m \log(mn))$: finding the minimal weight takes $O(\log m)$, and loop detection in F takes $O(\log n)$; in the worst case, all the selected edges form a loop with edges in F, and, therefore, this step has to be repeated for m times. The surface routing algorithm takes $O(n)$ to detect if the newly added edge intersects with any nodes. In this case, the loop iterates for n times, and therefore, its complexity is $O(mn + mn\log(mn) + n^2) = O(mn\log(mn))$, and since $m = n^2$, the complexity can be simplified as $O(n^3 \log n)$. The most expensive part is Lines 1–3 with the complexity $O(m^2) = O(n^4)$. Therefore, the total complexity of the non-duplicate case is $O(n^4)$.

For the worst case in which all the nodes duplicate for k times, the complexity of Line 6 is $O(kn^2)$. Line 7 has to repeat for n^2 times, which yields a complexity of $O(n^2 \log(kn^2) + n^2 \log n) = O(n^2 \log(kn^3))$. Lines 6–8 repeats for kn times, and, results in a total complexity of $O(kn^3 \log(kn^3))$ for the loop. Line 3 has a complexity of $O(k^2 n^4)$. Since the number of sets k is typically small, the complexity of the duplicate case is $O(n^4)$. In all, the time complexity of Algorithm 1 is $O(n^4)$.

2 Evaluation Results

In our paper, only part of the evaluation results are included due to the page limit. Here, we show results that are left out from the paper.

Summary statistics of the evaluation results of the overlap ratio of F2-Bubbles and Bubble Sets [2] are shown in Table 1. Whereas, summary statistics of the number of edge crossings of F2-Bubbles, Bubble Sets [2], Line Sets [1], Kelp Diagrams [3] and KelpFusion [4] are shown in Table 2, and total edge length of these methods are shown in Table 3. The associated boxplots are shown in Fig. 1. Full results of the evaluation are documented in Table 6.

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Table 1: Summary statistics of the overlap ratio.

Method	Type	Min	Q1	Median	Mean	Q3	Max
Ours	real	4.73	5.96	7.86	8.39	10.1	15.0
Ours	synthetic	1.1	3.06	4.81	5.82	7.92	13.4
BubbleSets	real	24.0	27.0	31.3	35.9	43.4	59.7
BubbleSets	synthetic	8.98	20.1	29.6	30.6	41.4	61.3

Table 2: Summary statistics of the number of edge crossings.

Method	Type	Min	Q1	Median	Mean	Q3	Max
Ours	real	4	12	17.5	19.6	18.8	56
Ours	synthetic	2	6.75	18	17.2	24.2	46
BubbleSets	real	8	35.2	39.5	44.9	53.8	102
BubbleSets	synthetic	7	18	38	44.1	72	97
KelpDiagrams	real	6	17.8	26	29.7	40	66
KelpDiagrams	synthetic	10	16.8	30	37.7	56	92
KelpFusion	real	9	31.2	43	50.7	71.8	107
KelpFusion	synthetic	20	26.8	42.5	65.2	94	161
LineSets	real	4	26.2	34	42.8	52.8	126
LineSets	synthetic	13	21.8	32	40.3	60	84

For real-world datasets, results of Wilcoxon tests show significant differences between F2-Bubbles and Bubble Sets on overlap ($V = 0, p = 0.001953$), the number of edge crossings ($V = 0, p = 0.005857$) and total edge length ($V = 2, p = 0.005859$), but difference on the number of bends is not significant ($V = 15.5, p = 0.7778$) ; significant differences between F2-Bubbles and KelpFusion on the number of edge crossings ($V = 0, p = 0.001953$), total edge length ($V = 0, p = 0.001953$), and the number of bends ($V = 0, p = 0.005857$); F2-Bubbles and Line Sets on the number of edge crossings ($V = 0, p = 0.009152$), total edge length ($V = 0, p = 0.001953$), and the number of bends ($V = 55, p = 0.001953$, and LineSets has fewer bends); F2-Bubbles and KelpDiagrams on the number of edge crossings ($V = 2, p = 0.01072$), total edge length ($V = 0, p = 0.001953$), and the number of bends ($V = 0, p = 0.005857$) .

Results of Wilcoxon test for synthetic datasets find significant differences between F2-Bubbles and Bubble Sets on overlap ($V = 0, p = 1.907 \times 10^{-6}$); significant differences between F2-Bubbles and Bubble Sets ($V = 0, p = 9.556 \times 10^{-5}$), F2-Bubbles and KelpFusion ($V = 0, p = 9.436 \times 10^{-5}$), F2-Bubbles and Line Sets ($V = 0, p = 9.516 \times 10^{-5}$) , F2-Bubbles and KelpDiagrams ($V = 0, p = 9.516 \times 10^{-5}$) for the number of edge crossings; significant differences between F2-Bubbles and Bubble Sets ($V = 0, p = 1.907 \times 10^{-6}$), F2-Bubbles and KelpFusion ($V = 0, p = 1.907 \times 10^{-6}$), F2-Bubbles and Line Sets ($V = 3, p = 9.537 \times 10^{-6}$) , F2-Bubbles and KelpDiagrams ($V = 0, p = 1.907 \times 10^{-6}$) for the total edge length; significant differences between F2-Bubbles and Bubble Sets ($V = 35, p = 0.02902$, Bubble Sets has shorter edge length), KelpFusion ($V = 0, p = 9.556 \times 10^{-5}$), KelpDiagrams($V = 0, p = 9.542 \times 10^{-5}$), and Line Sets ($V = 35, p = 0.02902$, Line Sets is significantly lower) for the number of bends.

3 Visualizations Used in the Evaluation

In this section, we include visualizations of techniques used in the evaluation that are left out from the paper. For each dataset, we compare results of our method to Bubble Sets, Line Sets, KelpFusion and KelpDiagrams visualizations. Since we evaluate only the edge crossings and the total edge length for Line Sets, KelpFusion and KelpDiagrams, we computed only the skeletons instead of the full visualizations of KelpFusion and KelpDiagrams due to its long computational time even for moderate-sized datasets (the scalability issue of KelpFusion has been pointed out by its creators [4]).

3.1 Real-World Data

We show results of real-world datasets used in the evaluation in Figs 2—11. The datasets are categorized into two groups: scatterplots (co2_gdppercapita, co2_income, life_children, life_income, and life_fertility) and geographical maps (Bronx, Brooklyn, Citywide, Manhattan, and Staten Island).

Scatterplots. Scatterplots are taken from the Gapminder Tools. Five datasets were used in the evaluation. Dataset co2_gdppercapita describes per capita carbon dioxide emissions from the fossil fuel consumption, cement production and gas flaring, minus export, plus import during the given year. Dataset co2_income describes carbon dioxide emissions per person and income per person. Dataset life_children describes the average

Table 3: Summary statistics of the total edge length.

Method	Type	Min	Q1	Median	Mean	Q3	Max
Ours	real	1428	2716	7007	6535	10745	11461
Ours	synthetic	5571	7223	8309	9618	12292	15255
BubbleSets	real	1754	3814	7821	7797	12519	13925
BubbleSets	synthetic	6275	8413	9232	10830	14232	17376
KelpDiagrams	real	1501	7448	10378	10084	14550	1691
KelpDiagrams	synthetic	2682	8791	10596	11287	14391	18778
KelpFusion	real	1449	8044	11194	10842	14641	18737
KelpFusion	synthetic	2406	9222	11348	11960	14970	20780
LineSets	real	1588	3334	8455	7791	12433	13886
LineSets	synthetic	6685	7821	10418	10612	12751	16180

Table 4: Summary statistics of the number of bends.

Method	Type	Min	Q1	Median	Mean	Q3	Max
Ours	real	28	66.8	114	120	185	212
Ours	synthetic	70	97.2	144	155	181	356
BubbleSets	real	30	69.8	120	133	213	247
BubbleSets	synthetic	73	103	150	155	189	356
KelpDiagrams	real	31	80.8	150	166	258	328
KelpDiagrams	synthetic	85	147	206	209	252	474
KelpFusion	real	31	108	199	214	344	383
KelpFusion	synthetic	94	185	247	264	335	601
LineSets	real	17	45.5	83	86	133	156
LineSets	synthetic	50	72.5	108	109	127	246

number of years a newborn child would live and the death of children under five years of age per 1,000 live births. Dataset life_income describes the average number of years a newborn child would live and income per person. Dataset life_fertility describes the average number of years a newborn child would live and the number of children that would be born to each woman with prevailing age-specific fertility rates. The results are shown in Figs. 3, 4, 7, 8 and 9.

Geographic Maps. The geographic map datasets are based on NYC Open Data[1]. The data sets we used are from the NYC Recovery Resiliency Projects Map which shows completed, ongoing, and planned recovery and resiliency projects throughout New York City. We extracted five datasets from the map according to the boroughs of New York, and for each data set we determined set relationship by dividing the points according to the status of the project. The results are shown in Figs. 2, 5, 6, 10 and 11.

3.2 Synthetic Data

We generate synthetic datasets using randomly placed set elements from a 2D uniform distribution for ex1 through ex13; on top of these, we limit the spatial spread of elements of each set for ex14 to ex20. The number of elements of each set is randomly generated within a range. Representative examples of synthetic datasets are shown in Figs. 12-21.

3.3 Different Parameters for Kelp Diagrams and KelpFusion

Kelp diagrams and KelpFusion do not have a default configuration, so we empirically set the parameters $b_t = 2$, $c_d = 1$, $c_\alpha = 100$, $c_I = 100$ for Kelp diagrams to generate proper links and t = 3 for KelpFusion and node radius r = 15 for both of them to achieve a "medium" effect that balances linear and areal regions. However, these parameters have the influence on the results. Here we present the results of Kelp Diagrams and KelpFusion with different parameters.

The boxplots in Fig. 22 to Fig. 27 show the summarized performance of Kelp Diagrams and KelpFusion on all datasets (red), and on real-world datasets (blue) with different parameters, where the corresponding visualizations of Kelp Diagrams(KD) are shown in Fig. 28 to Fig. 37 and the ones of KelpFusion(KF) are shown in Fig. 38 to Fig. 47.

Acknowledgments

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[1] https://data.cityofnewyork.us/City-Government/NYC-Recovery-Resiliency-Projects-Map-Points/kubx-z7zd

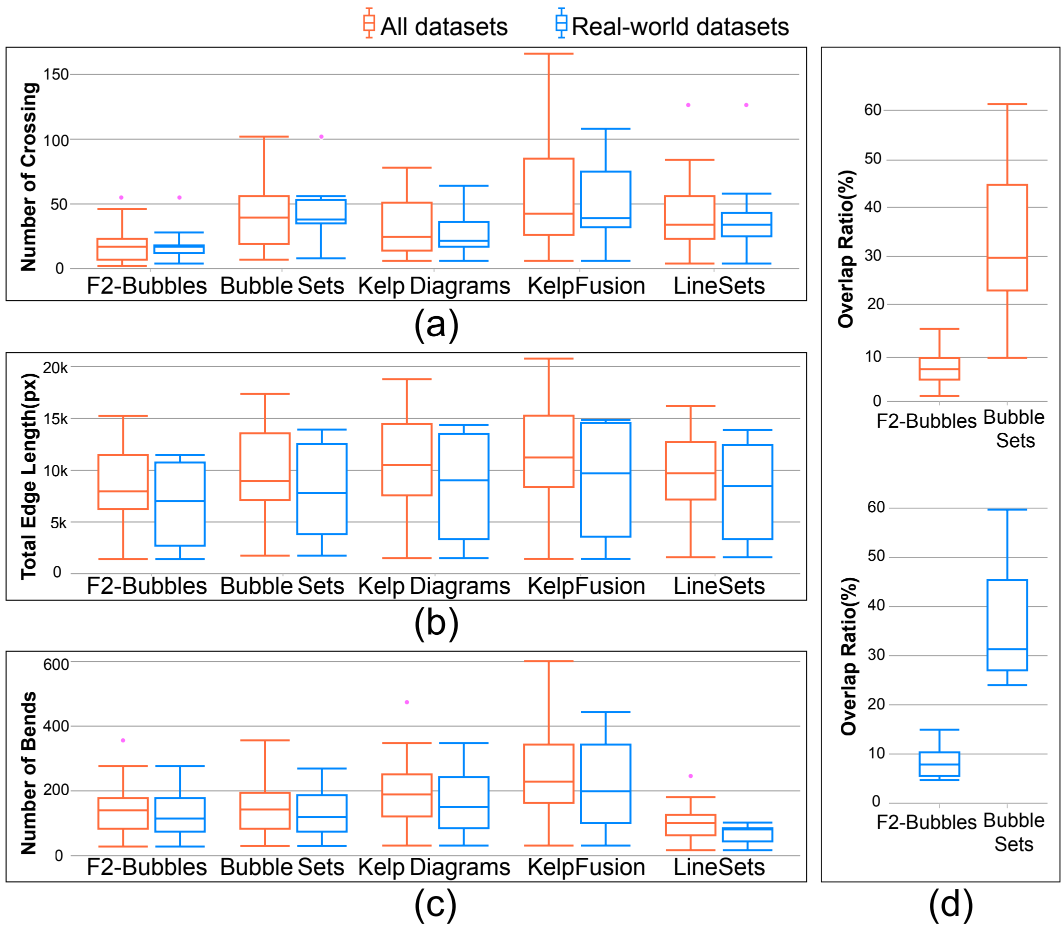
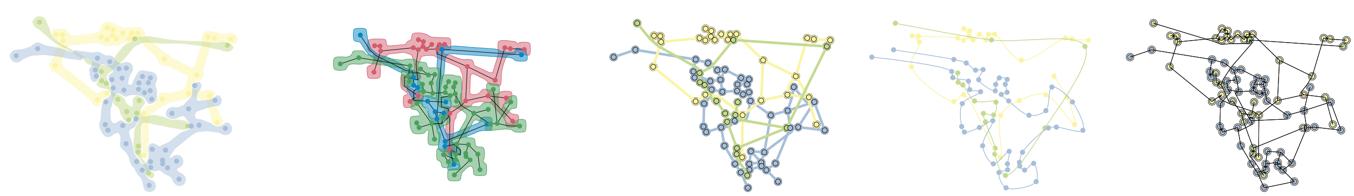
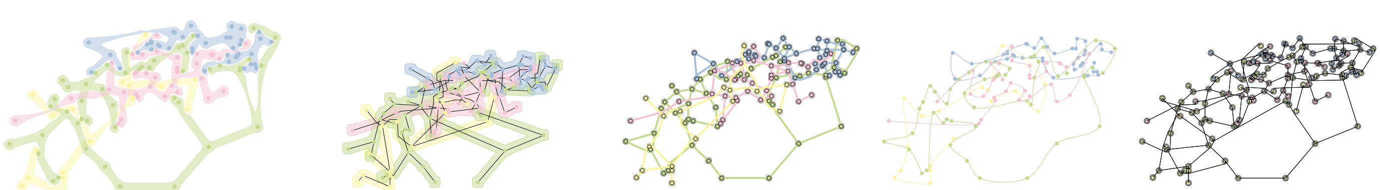


Fig. 1: Boxplots of evaluation performance of set overlay techniques on all datasets (red), and on real-world datasets (blue). The number of edge crossings are shown in (a), while total edge length are shown in (b), with overlap ratio in (c). Outliers are shown as points in the boxplots.



(a) Ours (b) Bubble Sets (c) KelpFusion (d) Line Sets (e) Kelp Diagrams

Fig. 2: Citywide (87 points)



(a) Ours (b) Bubble Sets (c) KelpFusion (d) Line Sets (e) Kelp Diagrams

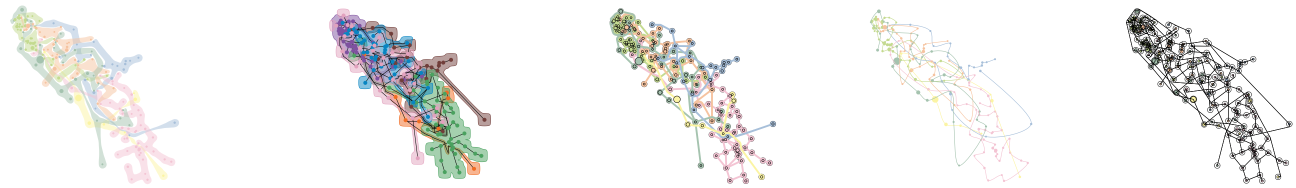
Fig. 3: Life Children (138 points)

Table 5: Comparisons of the overlap ratio (Overlap), the number of edge crossings (#Cross) of F2-Bubbles (Ours) and existing methods: Bubble Sets [2] (BS), Kelp Diagrams [3] (KD) KelpFusion [4] (KF), and Line Sets [1] (LS).

Dataset	#Elements	Ours-Overlap	BS-Overlap	Ours-#Cross	BS-#Cross	KF-#Cross	KD-#Cross	LS-#Cross
Bronx	25	7.39%	27.01%	4	8	9	6	4
Brooklyn	52	10.34%	37.35%	19	38	30	31	34
Citywide	87	7.16%	33.87%	12	26	35	20	25
co2_gdppercapita	93	5.56%	24.03%	18	41	44	24	43
co2_income	110	4.73%	27.12%	18	54	44	28	34
ex1	41	4.12%	9.96%	6	10	20	11	13
ex10	56	6.36%	12.41%	11	19	35	18	18
ex11	151	13.35%	49.13%	46	82	161	92	84
ex12	162	8.42%	40.36%	24	47	84	61	50
ex13	155	12.43%	50.55%	38	91	132	81	84
ex14	128	3.20%	21.65%	7	14	22	15	26
ex15	173	4.51%	47.96%	26	97	57	31	48
ex16	150	4.76%	37.60%	21	53	38	29	42
ex17	254	4.73%	61.29%	22	90	109	55	63
ex18	119	2.00%	23.44%	4	7	21	10	14
ex19	122	2.24%	29.56%	5	18	31	16	34
ex2	88	4.86%	18.19%	12	28	47	31	28
ex20	109	2.63%	31.81%	7	31	22	16	23
ex3	117	7.75%	44.63%	26	75	151	76	69
ex4	98	8.91%	38.60%	25	54	87	56	66
ex5	116	7.29%	29.66%	23	71	110	56	59
ex6	75	1.43%	12.92%	3	18	27	19	15
ex7	67	1.10%	8.98%	2	11	26	17	13
ex8	58	6.89%	20.73%	15	21	32	26	27
ex9	81	9.37%	22.87%	21	45	89	38	30
life_children	138	5.00%	28.76%	17	53	94	43	56
life_fertility	193	14.95%	59.70%	56	102	107	66	126
life_income	134	8.34%	26.72%	28	56	81	54	58
Manhattan	59	11.05%	48.54%	12	35	21	8	18
StatenIsland	59	9.35%	45.41%	12	36	42	17	30

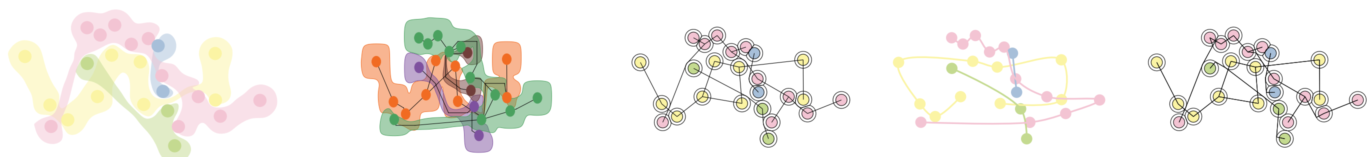
Table 6: Comparisons of the total edge length (#Length), the number of bends (#Bends) of F2-Bubbles (Ours) and existing methods: Bubble Sets [2] (BS), Kelp Diagrams [3] (KD) KelpFusion [4] (KF), and Line Sets [1] (LS).

Dataset	#Elements	Ours-#Length	BS-#Length	KD-#Length	KF-#Length	LS-#Length	Ours-#Bends	BS-#Bends	KD-#Bends	KF-#Bends	LS-#Bends
Bronx	25	1428.13	1753.65	1500.94	1561.99	1588.27	28	30	31	31	17
Brooklyn	52	3128.50	4118.54	4395.94	3565.54	3453.90	72	72	77	101	44
Citywide	87	6068.68	7265.97	8006.75	8592.77	7318.55	111	119	160	196	81
co2_gdppercapita	93	8744.77	8375.94	11292.37	11035.43	10755.88	118	120	141	202	85
co2_income	110	7945.63	10407.72	10962.96	10944.40	9590.63	159	153	202	268	102
ex1	41	6248.58	6312.42	7407.85	7320.71	7169.25	46	51	63	83	35
ex10	56	7961.99	8490.37	10287.28	11730.74	9082.39	65	69	92	130	50
ex11	151	13232.66	14869.57	17032.24	18432.25	14041.36	194	247	277	383	143
ex12	162	11765.30	13701.18	16338.36	15691.78	12554.42	212	233	328	370	156
ex13	155	13696.28	15783.19	17160.29	18237.84	14707.08	200	234	291	380	147
ex14	128	7837.78	8335.59	12010.40	11229.72	9801.39	153	162	251	303	120
ex15	173	7158.31	8574.05	9785.03	9049.58	6769.47	266	225	302	380	165
ex16	150	7775.70	8963.88	10412.66	9335.87	8446.81	208	194	257	332	142
ex17	254	12818.29	14762.00	16861.30	16245.86	12802.58	356	356	474	601	246
ex18	119	5570.90	6275.21	8325.93	7817.13	6992.70	141	147	189	237	111
ex19	122	6411.34	7119.88	9157.58	8703.49	7542.81	147	152	212	257	114
ex2	88	9661.98	10384.63	13052.95	13926.70	11534.91	109	113	148	190	82
ex20	109	5593.68	6352.59	7644.23	6744.38	6684.92	138	138	186	206	101
ex3	117	15255.12	17376.44	19843.32	20973.60	16180.36	153	160	217	290	109
ex4	98	11669.31	13558.02	16913.53	14987.91	12700.20	140	136	203	220	90
ex5	116	14905.35	16629.03	18417.38	19978.38	15468.98	157	164	210	277	108
ex6	75	7287.87	8946.73	11382.53	9962.85	8098.78	89	101	152	170	71
ex7	67	7701.51	8633.62	10926.02	11612.34	8530.73	79	83	121	163	63
ex8	58	8655.52	9500.83	11436.60	10436.88	11035.36	70	73	86	104	50
ex9	81	11154.98	12027.78	16086.51	17043.97	12105.46	100	104	145	205	73
life_children	138	10972.86	12519.37	15244.10	15168.11	12953.45	189	195	254	343	130
life_fertility	193	11461.27	13925.04	15187.75	13950.46	13885.87	277	269	348	444	181
life_income	134	10744.54	12811.90	14134.62	14713.94	12432.51	178	187	243	356	126
Manhattan	59	2143.75	2980.85	2752.04	2557.40	2591.82	83	75	88	94	51
StatenIsland	59	2715.77	3813.67	3332.94	3729.31	3334.41	74	74	85	105	51



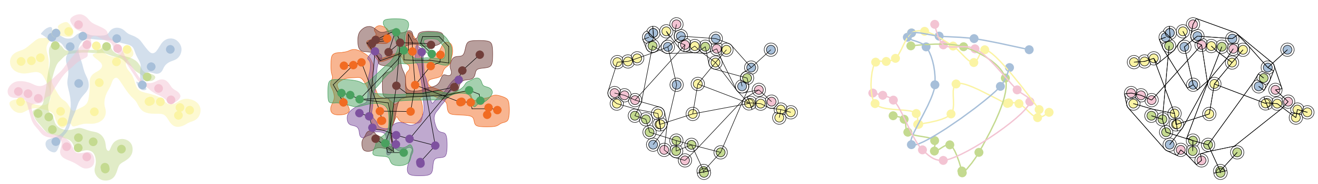
(a) Ours (b) Bubble Sets (c) KelpFusion (d) Line Sets (e) Kelp Diagrams

Fig. 4: Life Fertility (193 points)



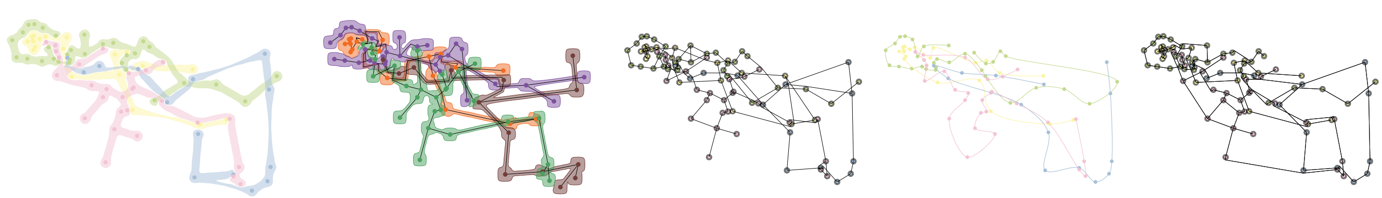
(a) Ours (b) Bubble Sets (c) KelpFusion (d) Line Sets (e) Kelp Diagrams

Fig. 5: Bronx (25 points)



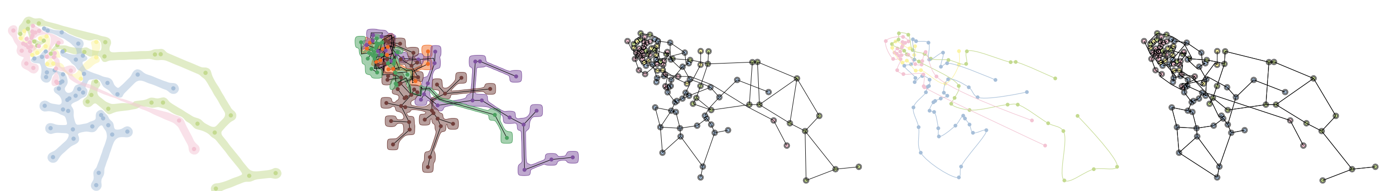
(a) Ours (b) Bubble Sets (c) KelpFusion (d) Line Sets (e) Kelp Diagrams

Fig. 6: Brooklyn (52 points)



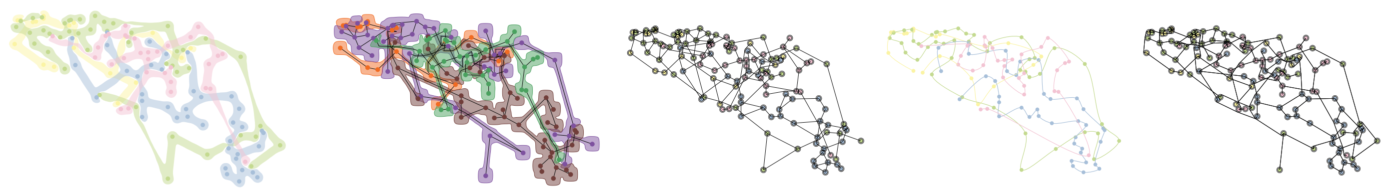
(a) Ours (b) Bubble Sets (c) KelpFusion (d) Line Sets (e) Kelp Diagrams

Fig. 7: co2-gdppercapita (93 points)



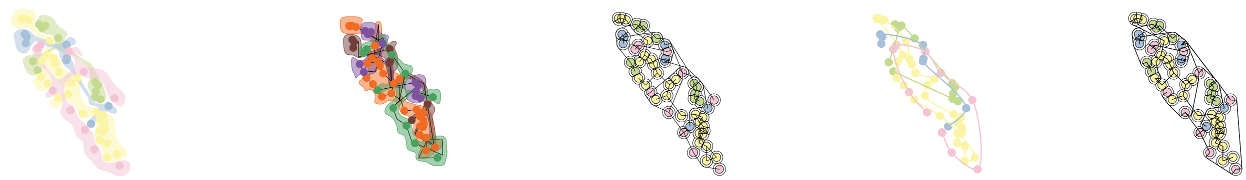
(a) Ours (b) Bubble Sets (c) KelpFusion (d) Line Sets (e) Kelp Diagrams

Fig. 8: co2-income (110 points)



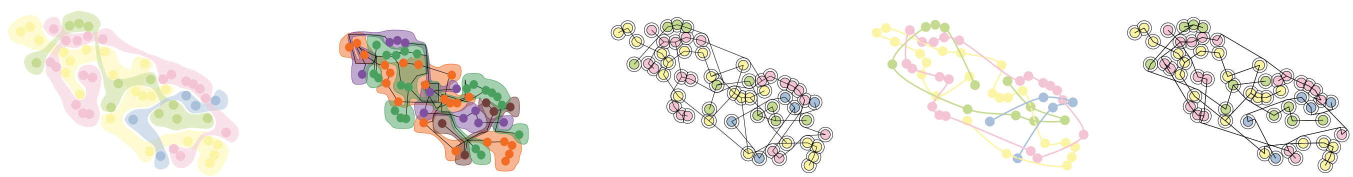
(a) Ours (b) Bubble Sets (c) KelpFusion (d) Line Sets (e) Kelp Diagrams

Fig. 9: Life Income (134 points)



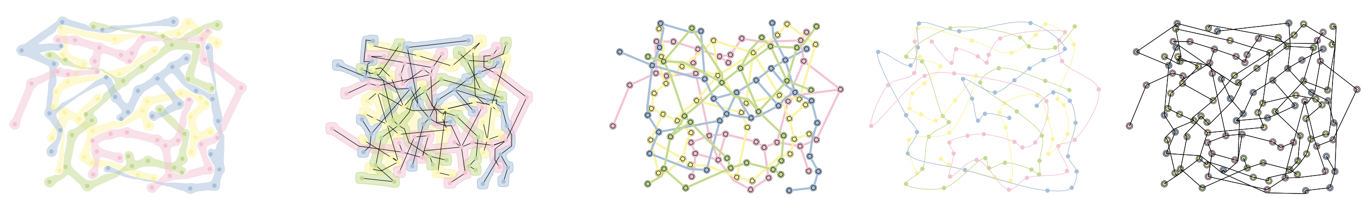
(a) Ours (b) Bubble Sets (c) KelpFusion (d) Line Sets (e) Kelp Diagrams

Fig. 10: Manhattan (59 points)



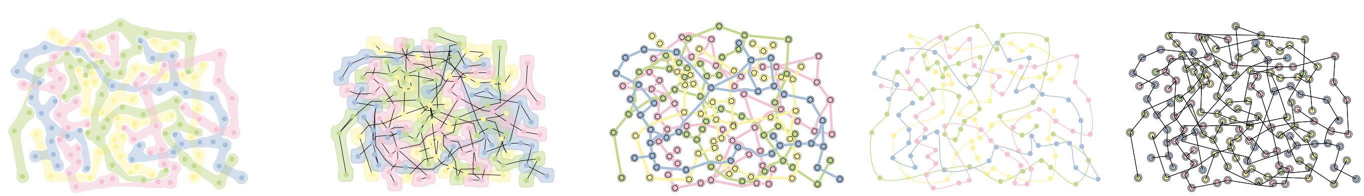
(a) Ours (b) Bubble Sets (c) KelpFusion (d) Line Sets (e) Kelp Diagrams

Fig. 11: Staten Island (59 points)



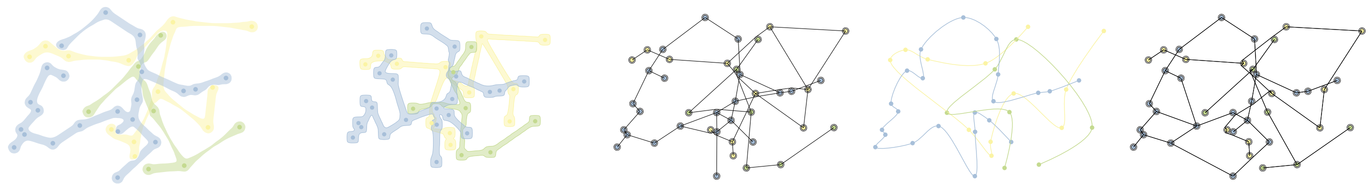
(a) Ours (b) Bubble Sets (c) KelpFusion (d) Line Sets (e) Kelp Diagrams

Fig. 12: EX3 (117 points)



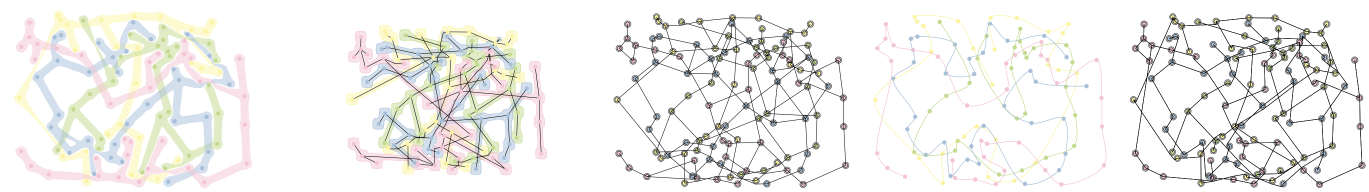
(a) Ours (b) Bubble Sets (c) KelpFusion (d) Line Sets (e) Kelp Diagrams

Fig. 13: EX13 (155 points)



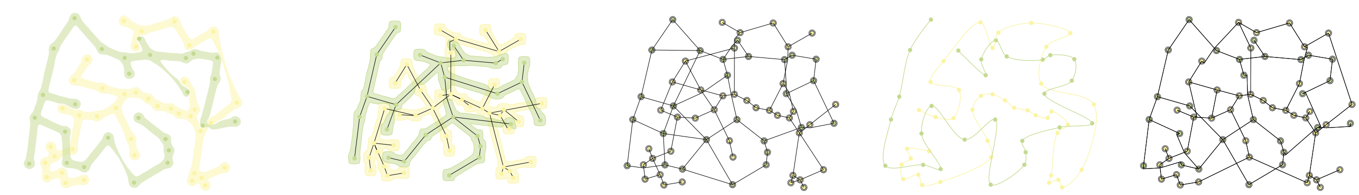
(a) Ours (b) Bubble Sets (c) KelpFusion (d) Line Sets (e) Kelp Diagrams

Fig. 14: EX1 (41 points)



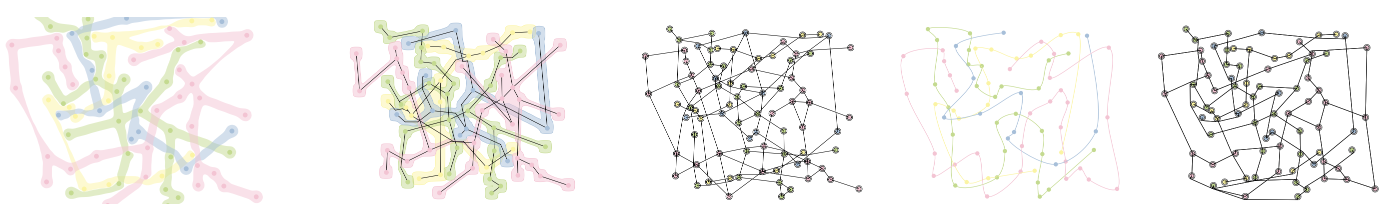
(a) Ours (b) Bubble Sets (c) KelpFusion (d) Line Sets (e) Kelp Diagrams

Fig. 15: EX5 (116 points)



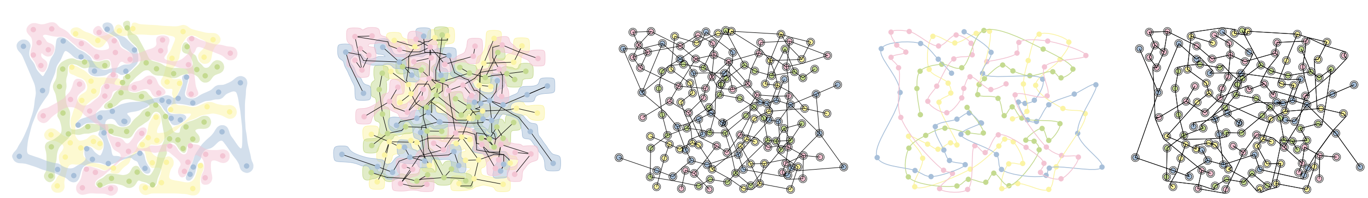
(a) Ours (b) Bubble Sets (c) KelpFusion (d) Line Sets (e) Kelp Diagrams

Fig. 16: EX7 (67 points)



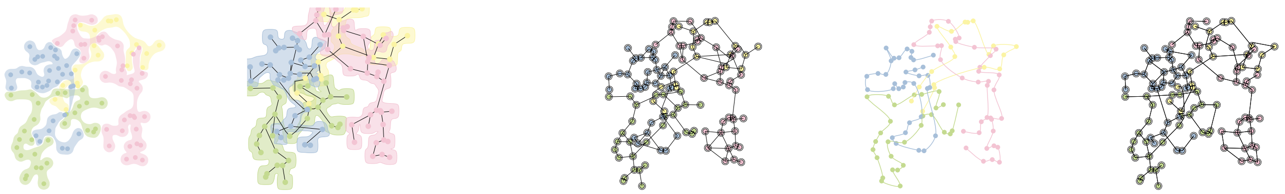
(a) Ours (b) Bubble Sets (c) KelpFusion (d) Line Sets (e) Kelp Diagrams

Fig. 17: EX9 (81 points)



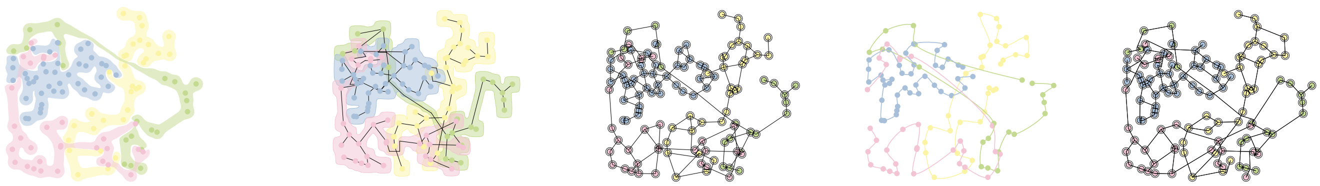
(a) Ours (b) Bubble Sets (c) KelpFusion (d) Line Sets (e) Kelp Diagrams

Fig. 18: EX11 (151 points)



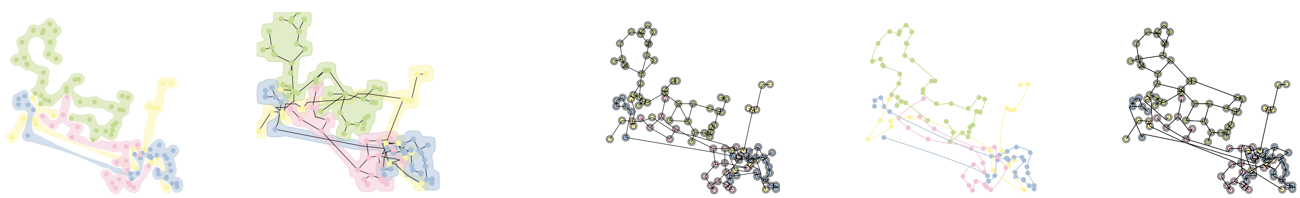
(a) Ours (b) Bubble Sets (c) KelpFusion (d) Line Sets (e) Kelp Diagrams

Fig. 19: EX18 (119 points)



(a) Ours (b) Bubble Sets (c) KelpFusion (d) Line Sets (e) Kelp Diagrams

Fig. 20: EX19 (122 points)



(a) Ours (b) Bubble Sets (c) KelpFusion (d) Line Sets (e) Kelp Diagrams

Fig. 21: EX20 (109 points)

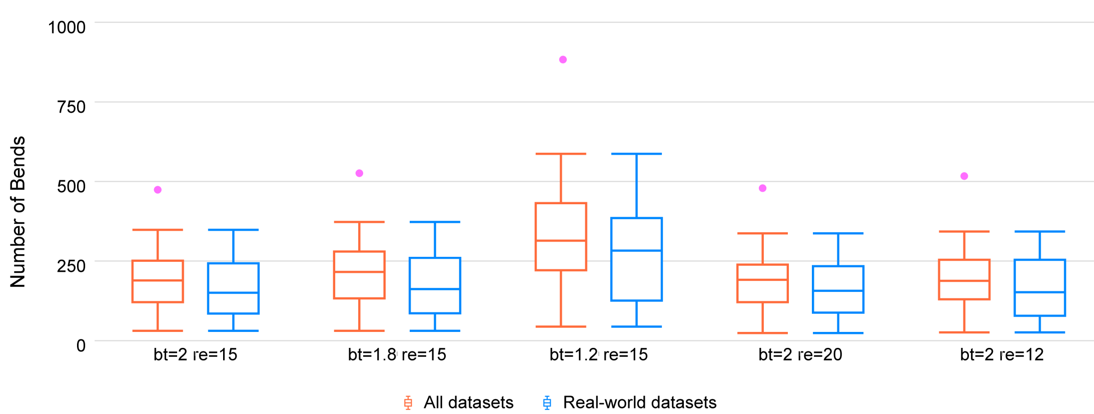


Fig. 22: Boxplots of the number of bends of Kelp Diagrams on all datasets (red), and on real-world datasets (blue). Outliers are shown as points in the boxplots.

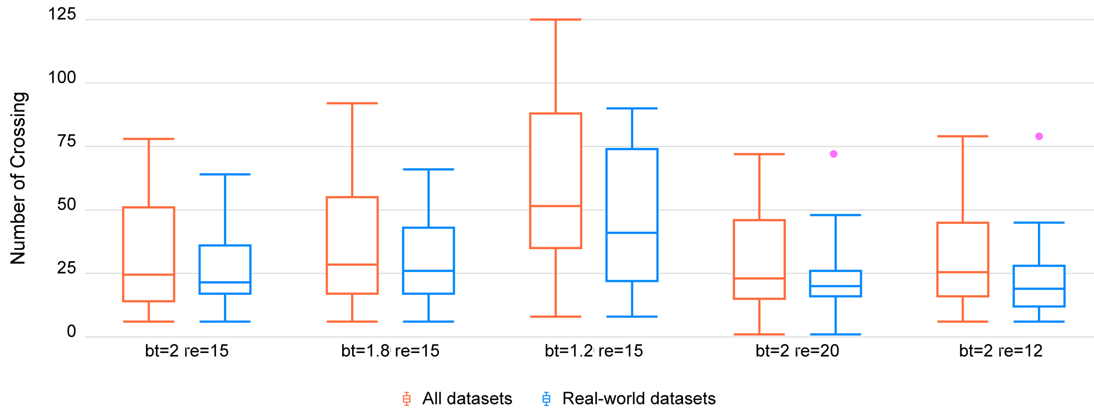


Fig. 23: Boxplots of the number of edge crossings of Kelp Diagrams on all datasets (red), and on real-world datasets (blue). Outliers are shown as points in the boxplots.

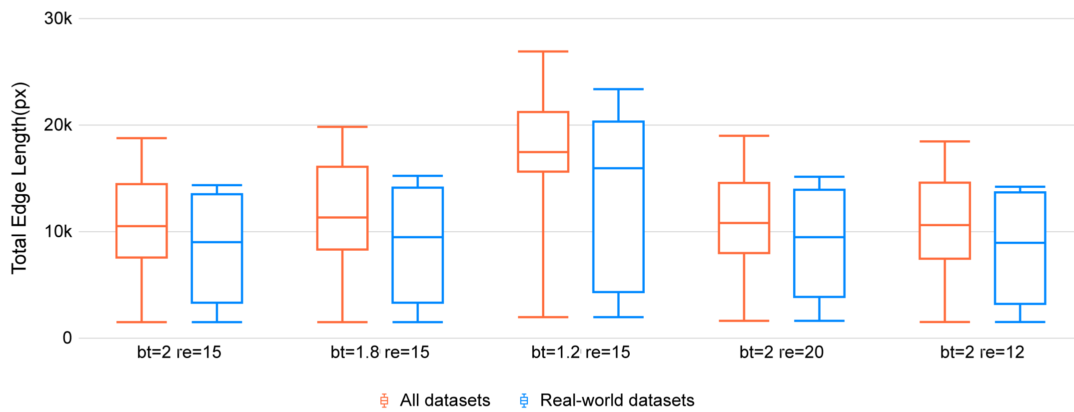


Fig. 24: Boxplots of the total edge length of Kelp Diagrams on all datasets (red), and on real-world datasets (blue).

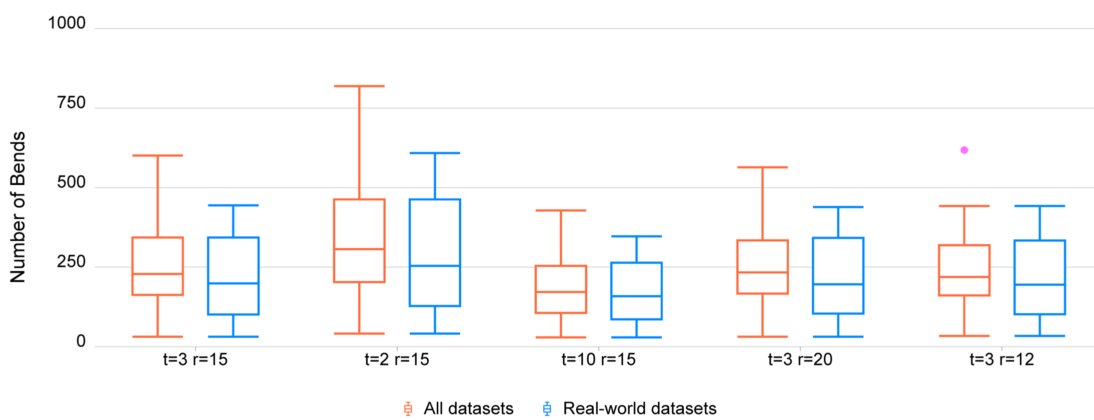


Fig. 25: Boxplots of the number of bends of KelpFusion on all datasets (red), and on real-world datasets (blue). Outliers are shown as points in the boxplots. Outliers are shown as points in the boxplots.

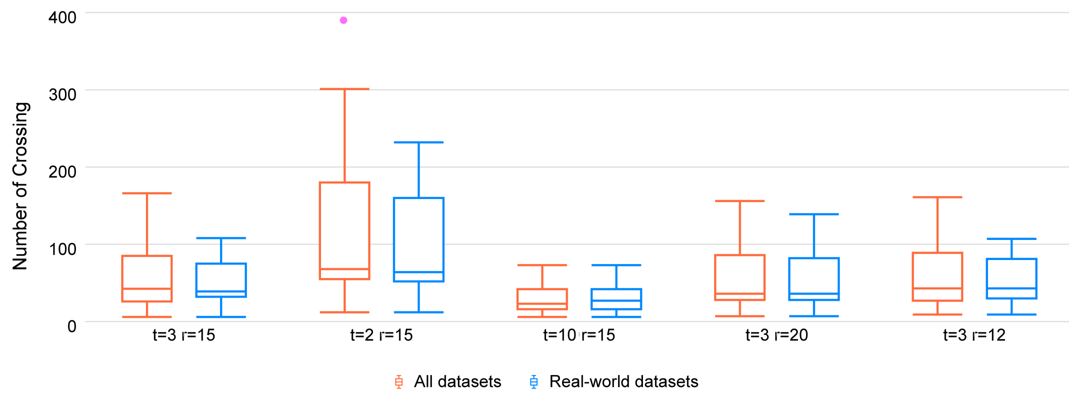


Fig. 26: Boxplots of the number of edge crossings of KelpFusion on all datasets (red), and on real-world datasets (blue). Outliers are shown as points in the boxplots. Outliers are shown as points in the boxplots.

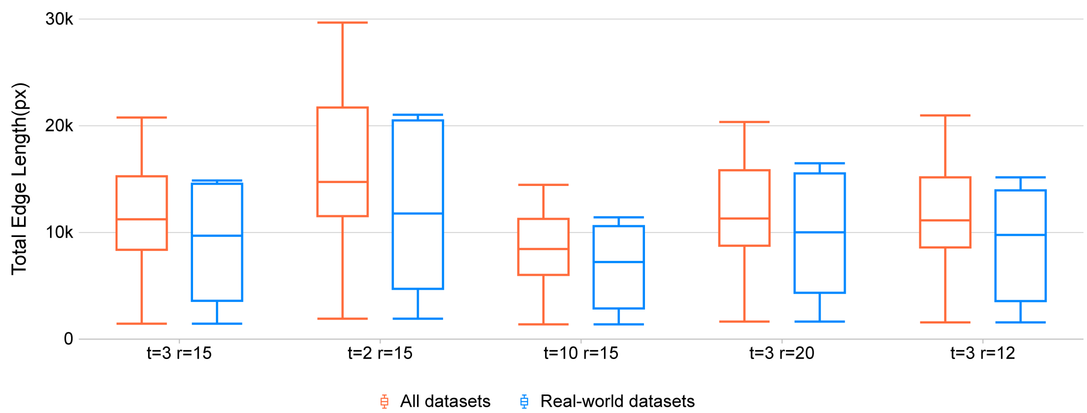
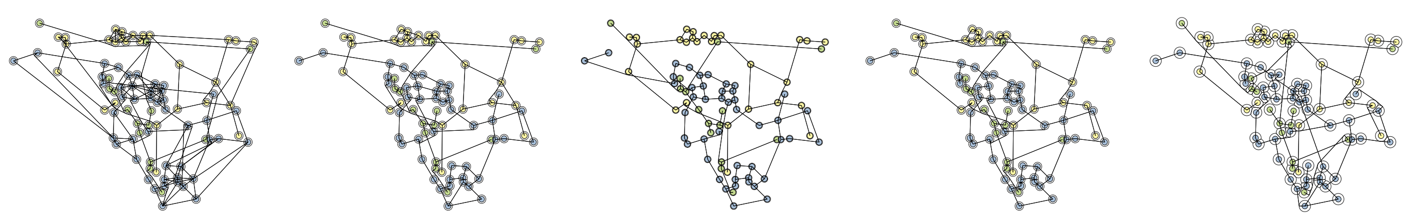
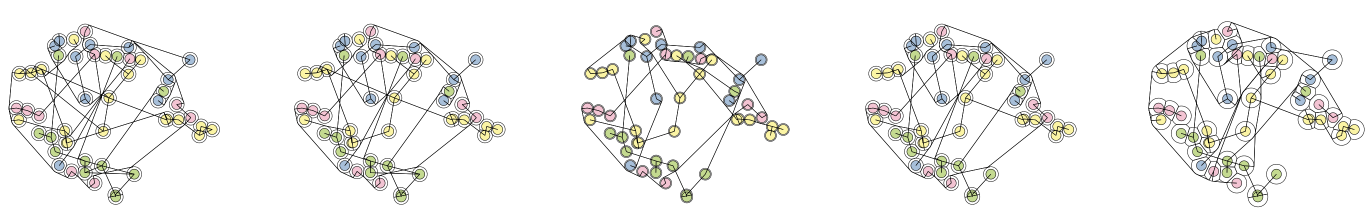


Fig. 27: Boxplots of the total edge length of KelpFusion on all datasets (red), and on real-world datasets (blue).



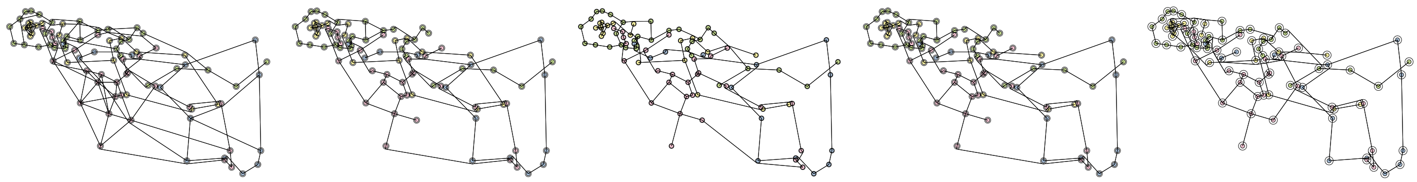
(a) KD bt=1.2 re=15 (b) KD bt=1.8 re=15 (c) KD bt=2 re=12 (d) KD bt=2 re=15 (e) KD bt=2 re=20

Fig. 28: Citywide (87 points)



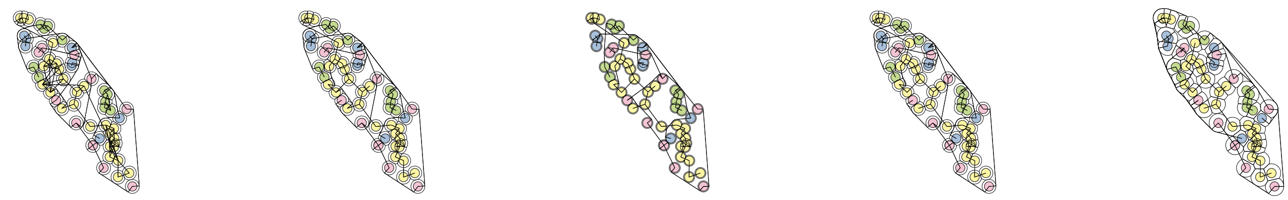
(a) KD bt=1.2 re=15 (b) KD bt=1.8 re=15 (c) KD bt=2 re=12 (d) KD bt=2 re=15 (e) KD bt=2 re=20

Fig. 29: Brooklyn (52 points)



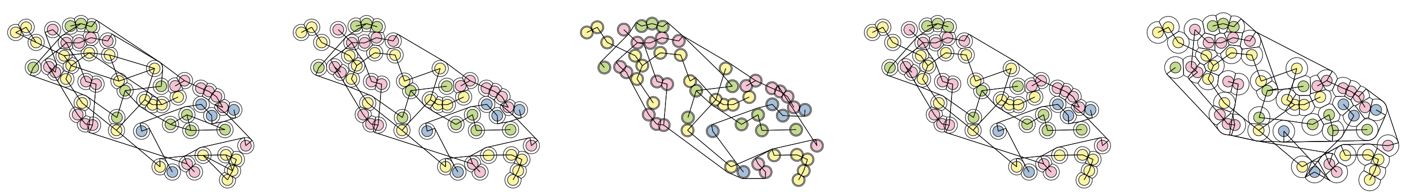
(a) KD bt=1.2 re=15 (b) KD bt=1.8 re=15 (c) KD bt=2 re=12 (d) KD bt=2 re=15 (e) KD bt=2 re=20

Fig. 30: co2-gdppercapita (93 points)



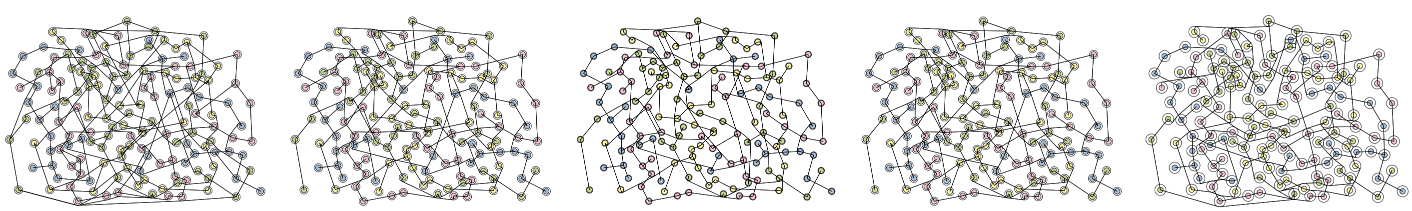
(a) KD bt=1.2 re=15 (b) KD bt=1.8 re=15 (c) KD bt=2 re=12 (d) KD bt=2 re=15 (e) KD bt=2 re=20

Fig. 31: Manhattan (59 points)



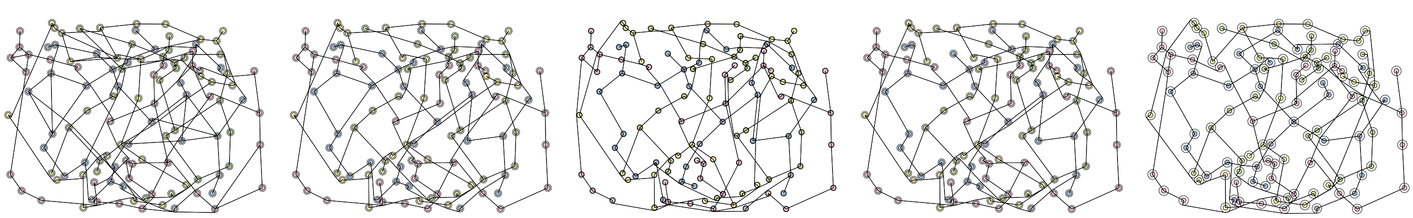
(a) KD bt=1.2 re=15 (b) KD bt=1.8 re=15 (c) KD bt=2 re=12 (d) KD bt=2 re=15 (e) KD bt=2 re=20

Fig. 32: Staten Island (59 points)



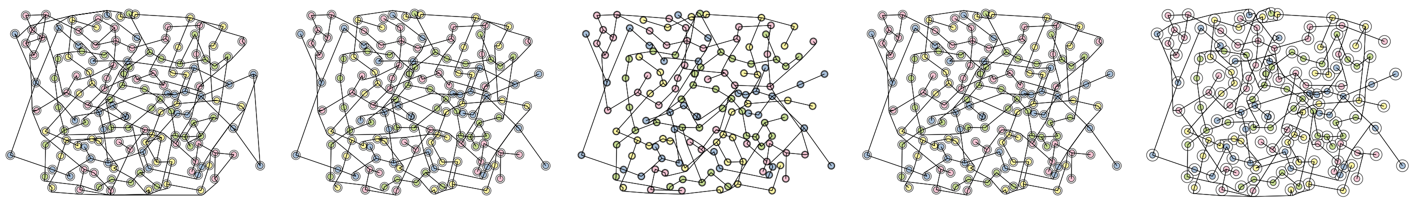
(a) KD bt=1.2 re=15 (b) KD bt=1.8 re=15 (c) KD bt=2 re=12 (d) KD bt=2 re=15 (e) KD bt=2 re=20

Fig. 33: EX13 (155 points)



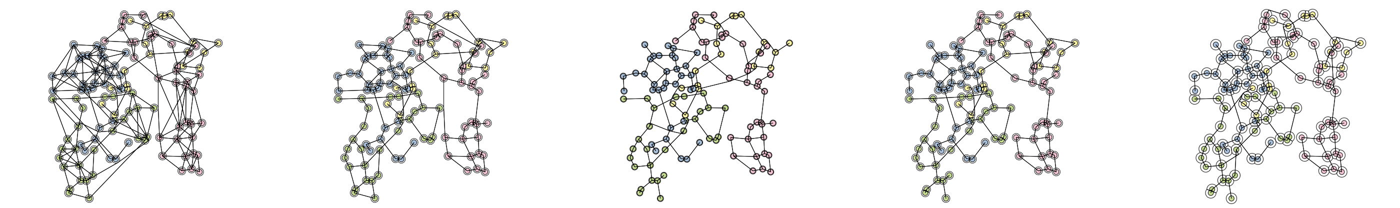
(a) KD bt=1.2 re=15 (b) KD bt=1.8 re=15 (c) KD bt=2 re=12 (d) KD bt=2 re=15 (e) KD bt=2 re=20

Fig. 34: EX5 (116 points)



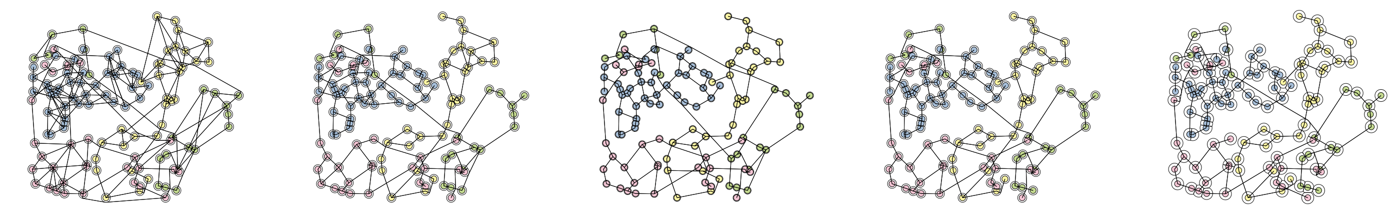
(a) KD bt=1.2 re=15 (b) KD bt=1.8 re=15 (c) KD bt=2 re=12 (d) KD bt=2 re=15 (e) KD bt=2 re=20

Fig. 35: EX11 (151 points)



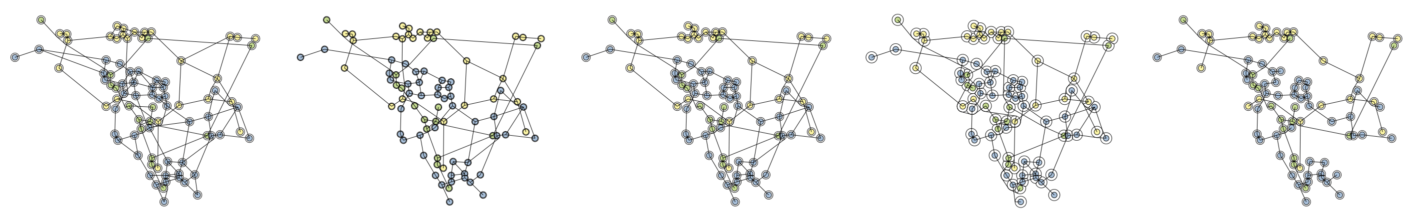
(a) KD bt=1.2 re=15 (b) KD bt=1.8 re=15 (c) KD bt=2 re=12 (d) KD bt=2 re=15 (e) KD bt=2 re=20

Fig. 36: EX18 (119 points)



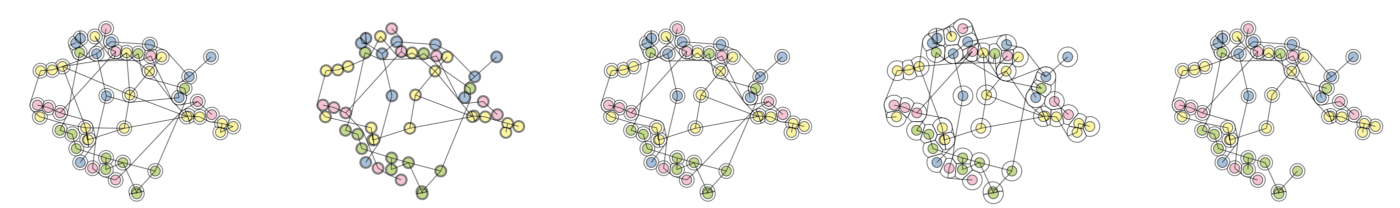
(a) KD bt=1.2 re=15 (b) KD bt=1.8 re=15 (c) KD bt=2 re=12 (d) KD bt=2 re=15 (e) KD bt=2 re=20

Fig. 37: EX19 (122 points)



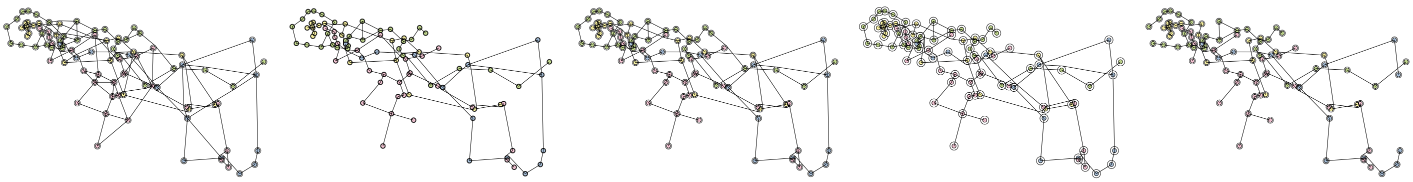
(a) KF t=2 re=15 (b) KF t=3 re=12 (c) KF t=3 re=15 (d) KF t=3 re=20 (e) KF t=10 re=15

Fig. 38: Citywide (87 points)



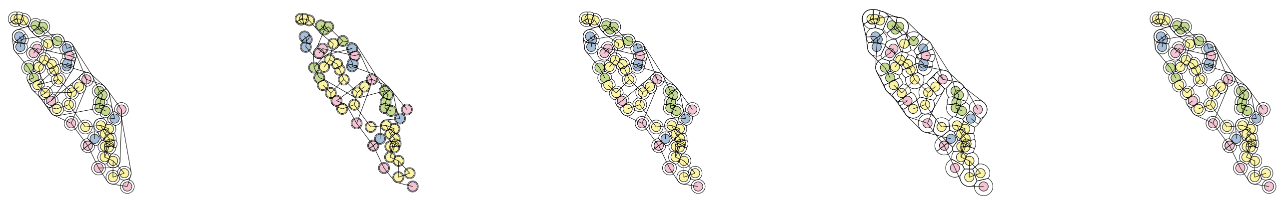
(a) KF t=2 re=15 (b) KF t=3 re=12 (c) KF t=3 re=15 (d) KF t=3 re=20 (e) KF t=10 re=15

Fig. 39: Brooklyn (52 points)



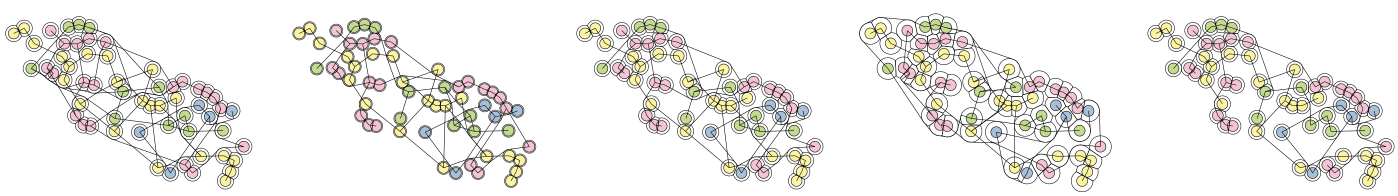
(a) KF t=2 re=15 (b) KF t=3 re=12 (c) KF t=3 re=15 (d) KF t=3 re=20 (e) KF t=10 re=15

Fig. 40: co2-gdppercapita (93 points)



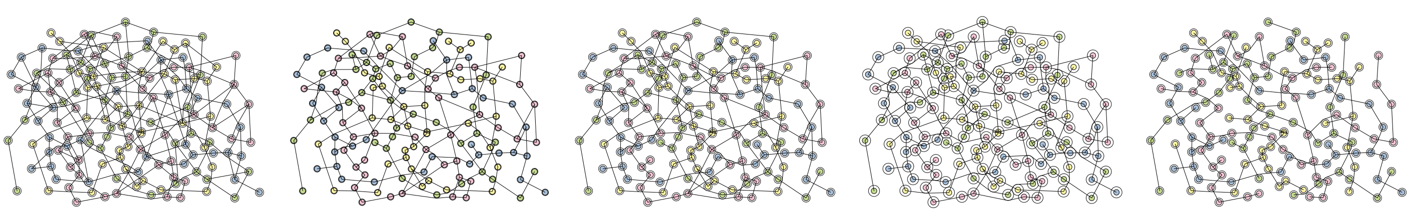
(a) KF t=2 re=15 (b) KF t=3 re=12 (c) KF t=3 re=15 (d) KF t=3 re=20 (e) KF t=10 re=15

Fig. 41: Manhattan (59 points)



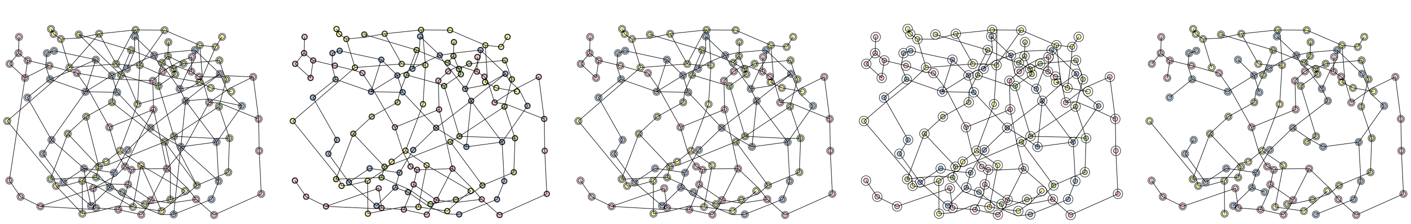
(a) KF t=2 re=15 (b) KF t=3 re=12 (c) KF t=3 re=15 (d) KF t=3 re=20 (e) KF t=10 re=15

Fig. 42: Staten Island (59 points)



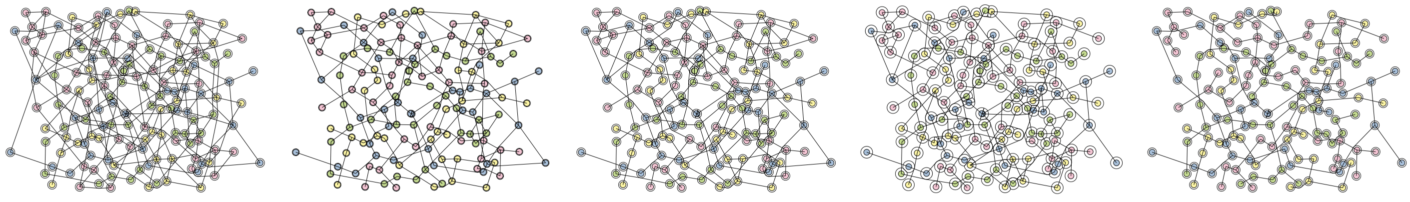
(a) KF t=2 re=15 (b) KF t=3 re=12 (c) KF t=3 re=15 (d) KF t=3 re=20 (e) KF t=10 re=15

Fig. 43: EX13 (155 points)



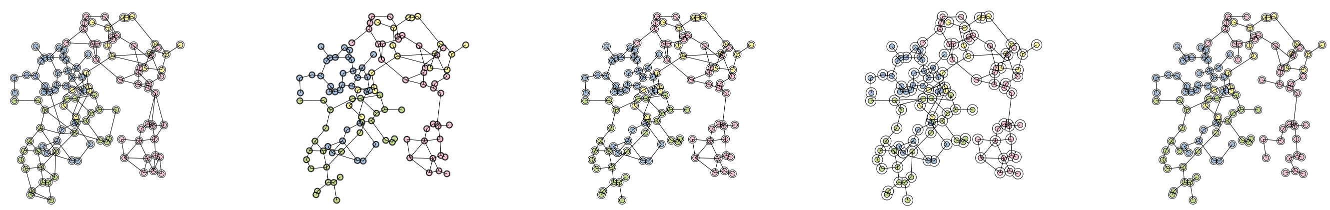
(a) KF t=2 re=15 (b) KF t=3 re=12 (c) KF t=3 re=15 (d) KF t=3 re=20 (e) KF t=10 re=15

Fig. 44: EX5 (116 points)



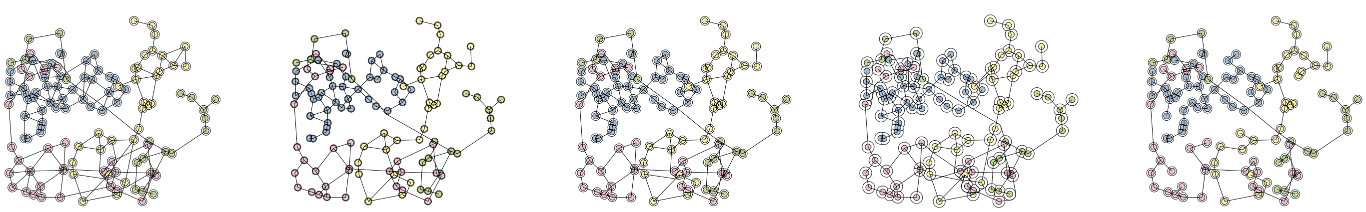
(a) KF t=2 re=15 (b) KF t=3 re=12 (c) KF t=3 re=15 (d) KF t=3 re=20 (e) KF t=10 re=15

Fig. 45: EX11 (151 points)



(a) KF t=2 re=15 (b) KF t=3 re=12 (c) KF t=3 re=15 (d) KF t=3 re=20 (e) KF t=10 re=15

Fig. 46: EX18 (119 points)



(a) KF t=2 re=15 (b) KF t=3 re=12 (c) KF t=3 re=15 (d) KF t=3 re=20 (e) KF t=10 re=15

Fig. 47: EX19 (122 points)

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