# Image-Based Aspect Ratio Selection

# - Supplementary Material -

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This supplemental material file provides additional experimental and evaluation results for our submitted paper titled "Image-Based Aspect Ratio Selection." In summary, it includes the following four parts:

- First, we provided the detailed derivations of imgAWO and imgRV (Section 1).
- Hereafter, we employed all six aspect ratio selection methods discussed in our paper (i.e., imgRV, imgAL, imgAWO, isoRV, isoAL, and isoAWO) and compared their performance on both deviations (see main paper for its definition) and the method runtime (Section 2).
- Furthermore, we conducted a complete comparison between imgRV and the six criterions presented in [5] using all the datasets appeared in their paper and supplemental material [5] (Section 3).
- Finally, we presented additional results produced from our method (Section 4).

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#### 1 DERIVING IMGAWO AND IMGRV

In this appendix, we present the details for deriving the formulations for imgAWO and imgRV:

**ImgAWO.** Since Eq. (7) is not a single line integral as for AL, we cannot directly apply Eq. (6) to Eq. (7). However, by considering C in each line integral term (see the numerator and denominator in Eq. (7)) as the set of all isolines over all isovalues, i.e.,  $C = \{C_t\} \ \forall t \in [0,1]$ , we can rewrite the numerator and denominator in Eq. (7) as

$$\min_{\alpha \in (0,\infty)} \left| \frac{\int_0^1 \int_{C_t} |\theta(\alpha)| \, \mathrm{d}s \, \mathrm{d}t}{\int_0^1 \int_{C_t} \, \mathrm{d}s \, \mathrm{d}t} - \frac{\pi}{4} \right| , \tag{1}$$

Gradient,

(6).

and then apply Eq. (6) to each line integral term by putting  $f = |\theta(\alpha)|$  and f = 1 into Eq. For the line integral with f = 1, the right hand side of Eq. (6) can be easily obtained. For the line integral with  $f = |\theta(\alpha)|$ , we consider the local tangent vector in  $\Omega_{\alpha}$ , i.e.  $\begin{pmatrix} 1 & \partial \rho & \partial \rho \\ \partial \rho & \partial \rho \end{pmatrix}$ , which is always perpendicular to the local gradient vector from  $\frac{\partial \rho}{\partial \alpha} = \frac{\partial \rho}{\partial \alpha} =$ 

i.e.,  $(\frac{\partial \rho}{\partial x}, -\frac{\partial \rho}{\partial x})$ , which is always perpendicular to the local gradient vector from Eq. (5); see the inset figure on the right. Hence, the local orientation (tangent)  $\theta$  along the isoline can be expressed as:

$$\theta = \tan^{-1} \left( \frac{-\frac{\partial \rho}{\partial x}}{\frac{1}{\alpha} \frac{\partial \rho}{\partial y}} \right)$$
, so  $|\theta| = \tan^{-1} \left( \alpha \frac{\partial \rho}{\partial x} / \frac{\partial \rho}{\partial y} \right)$ . (2)

Putting the above results and Eq. (7) together, we can obtain the objective of imgAWO as shown in Eq. (8).

**ImgRV.** Given the line integral form of RV presented in Eq. (9), we, again, consider  $C = \{C_t\} \ \forall t \in [0, 1]$  and rewrite Eq. (9) as

$$\alpha = \frac{\int_0^1 \int_{C_t} |\cos(\theta)| \, \mathrm{d}s \, \mathrm{d}t}{\int_0^1 \int_{C_t} |\sin(\theta)| \, \mathrm{d}s \, \mathrm{d}t}. \tag{3}$$

We can then apply Eq. (6) to each line integral in the numerator and denominator, and obtain

$$\alpha = \frac{\int_{\Omega_{1,0}} |\cos(\theta)| ||(\frac{\partial \rho}{\partial x}, \frac{\partial \rho}{\partial y})|| \, dx \, dy}{\int_{\Omega_{1,0}} |\sin(\theta)| \, ||(\frac{\partial \rho}{\partial x}, \frac{\partial \rho}{\partial y})|| \, dx \, dy}. \tag{4}$$

By further considering the local gradient  $(\frac{\partial \rho}{\partial x}, \frac{\partial \rho}{\partial y})$  and its corresponding tangent vector  $(\frac{\partial \rho}{\partial y}, -\frac{\partial \rho}{\partial x})$  in  $\Omega_{1.0}$ , we can simplify Eq. (4), and obtain the formulation of imgRV as presented in Eq. (10).

In addition, there are the equations (presented in our paper) mentioned above:

$$\nabla \rho_{\alpha}(\mathbf{x}) = \left(\frac{\partial \rho}{\partial x}, \frac{1}{\alpha} \frac{\partial \rho}{\partial y}\right),\tag{5}$$

$$\int_{0}^{1} \left( \int_{L_{\alpha}(t)} f(\mathbf{X}, \alpha) \, \mathrm{d}s \right) \, \mathrm{d}t = \int_{\Omega_{1.0}} f(\mathbf{X}, \alpha) \left| \left| \left( \alpha \frac{\partial \rho}{\partial x}, \frac{\partial \rho}{\partial y} \right) \right| \right| \, \mathrm{d}x \, \mathrm{d}y, \tag{6}$$

$$\min_{\alpha \in (0,\infty)} \left| \frac{\int_C |\theta(\alpha)| \mathrm{d}s}{\int_C \mathrm{d}s} - \frac{\pi}{4} \right| . \tag{7}$$

$$\min_{\alpha \in (0,\infty)} \left| \frac{\int_{D} \left| \tan^{-1} \left( \frac{\partial \rho}{\partial x} / \frac{\partial \rho}{\alpha \partial y} \right) \right| \left| \left| \left( \alpha \frac{\partial \rho}{\partial x}, \frac{\partial \rho}{\partial y} \right) \right| \right| dx dy}{\int_{D} \left| \left| \left( \alpha \frac{\partial \rho}{\partial x}, \frac{\partial \rho}{\partial y} \right) \right| \right| dx dy} - \frac{\pi}{4} \right|, \tag{8}$$

$$\alpha = \frac{\sum_{i} |\Delta x_{i}|}{\sum_{i} |\Delta y_{i}|} = \frac{\int_{C} |\cos \theta| \, ds}{\int_{C} |\sin \theta| \, ds}. \tag{9}$$

$$\alpha = \frac{\int_{\Omega_{1,0}} ||\frac{\partial \rho}{\partial y}|| dx dy}{\int_{\Omega_{1,0}} ||\frac{\partial \rho}{\partial x}|| dx dy},$$
(10)

#### 2 COMPREHENSIVE COMPARISON WITH MORE ASPECT RATIO SELECTION METHODS

The overall results of the deviation measure comparing with isoRV for all the six methods are shown in Figure 1(a). Note that the number of isovalues (m) is 1000 for isoline-based methods, if we want to ensure high precision. We generate the boxplots by applying the 100 datasets described in Section 6.1 of our paper. It is obvious to show that our imgRV method selects the most similar result with isoRV compared to all other methods. Figure 1(b) summarizes and compares the time performance of the six methods, where m = 500 is used here for keeping the consistency with the paper; this result clearly shows that imgRV is the fastest method. Furthermore, more details can be found in Appendices A and B for all dataset comparisons between the methods with m set to be 100 and 500, respectively.

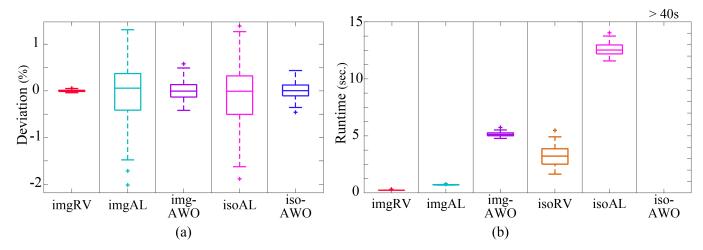


Fig. 1: Comprehensive comparison of the six methods. (a) Deviation of aspect ratios selected by our image-based methods (imgRV, imgAL, and imgAWO) and two isoline-based methods (isoAL and isoAWO, with 1000 isovalues) from those selected by isoRV (with 1000 isovalues); and (b) runtime of our methods and the three isoline-based methods (with 500 isovalues) for aspect ratio selection.

# 3 COMPARISON WITH ALL SIX CRITERIA IN FINK ET AL.

Figures 2, 3, and 4 show the comparison with more datasets [3, 11, 7, 6, 10, 5].

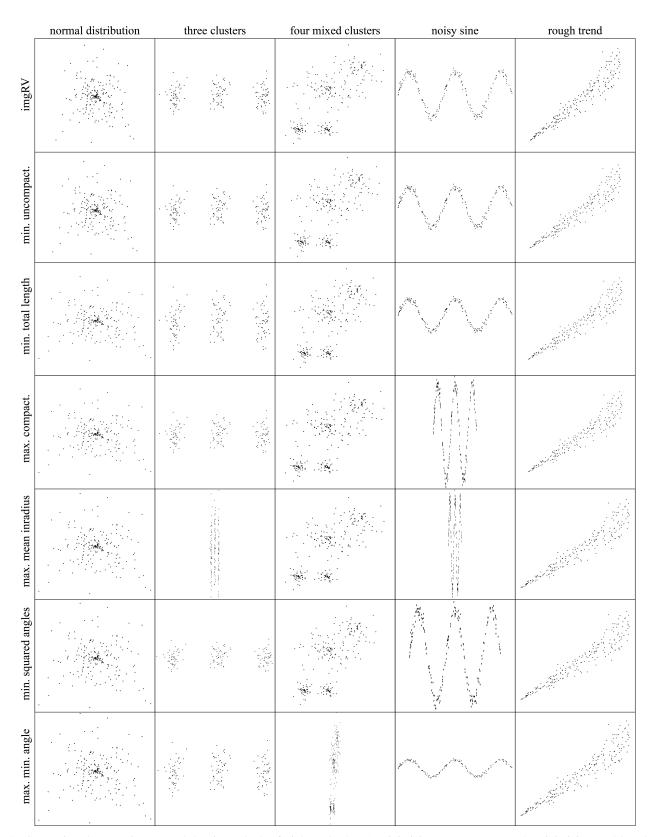


Fig. 2: Comparison between imgRV and the six methods of Fink et al. [5]: (1) minimizing uncompactness, (2) minimizing total length, (3) maximizing compactness, (4) maximizing mean inradius, (5) minimizing squared angles, and (6) maximizing minimum angle, over the five synthesized datasets.

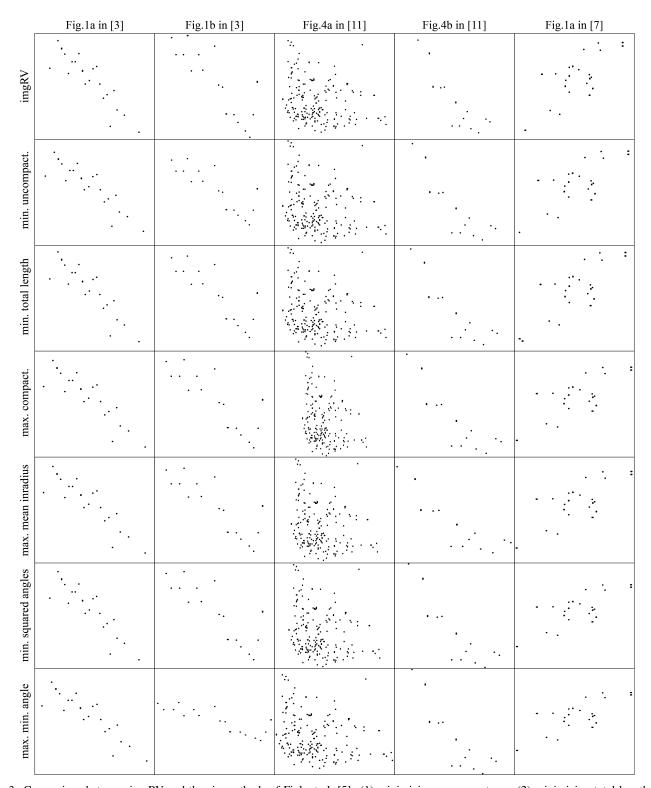


Fig. 3: Comparison between imgRV and the six methods of Fink et al. [5]: (1) minimizing uncompactness, (2) minimizing total length, (3) maximizing compactness, (4) maximizing mean inradius, (5) minimizing squared angles, and (6) maximizing minimum angle, over five datasets from *Nature* papers [3, 11, 7].

	Fig.1b in [7]	Fig.2a in [7]	Fig.2b in [7]	Fig.1d in [6]	Fig.7 in [10]
imgRV					
min. uncompact.					
min. total length					
max. compact.					
max. mean inradius					
min. squared angles					
max. min. angle					

Fig. 4: Comparison between imgRV and the six methods of Fink et al. [5]: (1) minimizing uncompactness, (2) minimizing total length, (3) maximizing compactness, (4) maximizing mean inradius, (5) minimizing squared angles, and (6) maximizing minimum angle, over five datasets from *Nature* and *TVCG* papers [7, 6, 10].

#### 4 ADDITIONAL RESULTS FROM OUR METHOD

Additional results are provided in this section.

# 4.1 Region-of-Interest (ROI) Banking

Figures 5 and 6 present two ROI banking results.

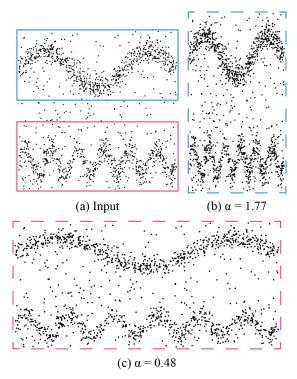


Fig. 5: ROI banking on the scatterplot with two different sine-trend clusters and random noise points. Our method can take user-specified ROIs (red and blue boxes in (a)) as inputs and select aspect ratios (b, c) that focus on revealing patterns inside.

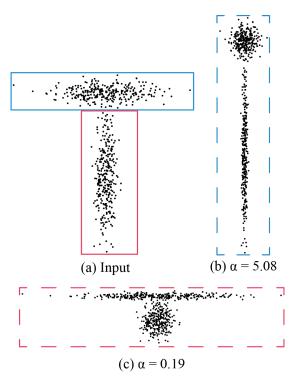


Fig. 6: ROI banking on the scatterplot of two Gaussian clusters. Our method can take user-specified ROIs (red and blue boxes in (a)) as inputs and select aspect ratios (b, c) that focus on revealing patterns inside.

#### 4.2 Multi-Scale Banking

Figure 7 shows another instance of our multi-scale banking result with the Twosquare dataset [9].

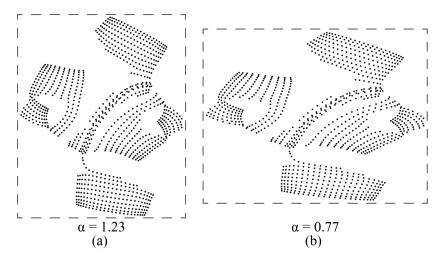


Fig. 7: Multi-scale banking on the Twosquare dataset [9]. Two aspect ratios are picked to show patterns of interest at different scales.

# 4.3 Image Data

Moreover, we generate more results by selecting aspect ratios for different images from internet in Figure 8 for the typical temperature dataset at Shawnigan Lake [8] and Figure 9 for the daily average temperatures dataset of New York, San Francisco and Austin [2].

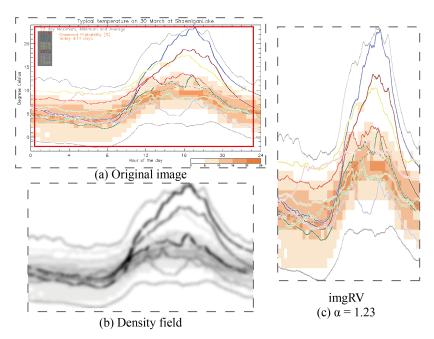


Fig. 8: Selecting a good aspect ratio for an image from the Internet [8]: (a) crop a relevant image part; (b) density field converted from the cropped image; and (c) result plotted with our selected aspect ratio.

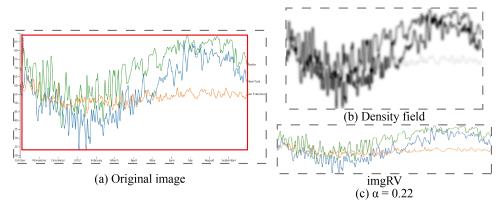


Fig. 9: Selecting a good aspect ratio for an image from the Internet [2]: (a) crop a relevant image part; (b) density field converted from the cropped image; and (c) result plotted with our selected aspect ratio.

# 4.4 Density Function Plots

Finally, three examples of density function plots are presented in Figure 10 for the *blunt-fin* dataset [1], Figure 11 for the *Turbulent* dataset [12], and Figure 12 for the *Feet* dataset [12].

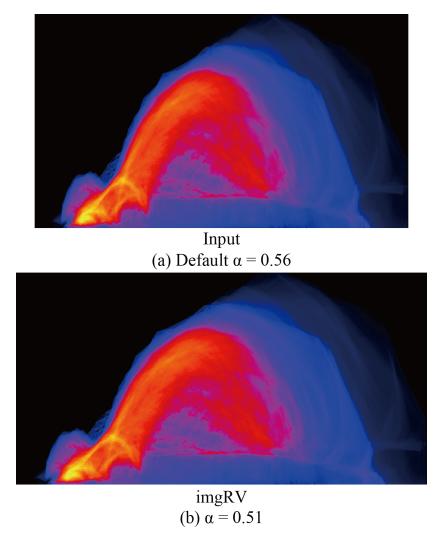


Fig. 10: Selecting an aspect ratio directly from a density function plot. (a) The input dataset is a continuous scatterplot of the *blunt-fin* dataset [1]. (b) Rescaled using a selected by imgRV.

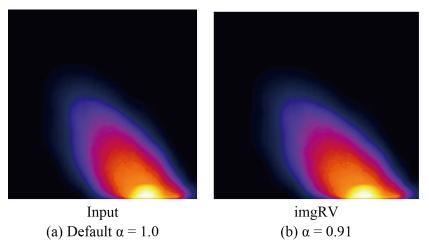


Fig. 11: Selecting an aspect ratio directly from a density function plot. (a) The input dataset is a continuous scatterplot of the *Turbulent* dataset [12]. (b) Rescaled using a selected by imgRV.

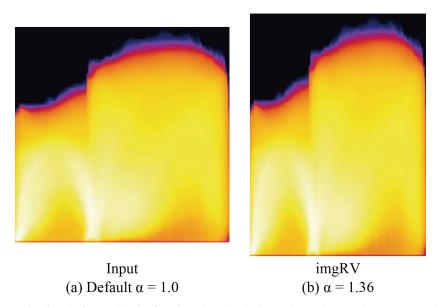


Fig. 12: Selecting an aspect ratio directly from a density function plot. (a) The input dataset is a continuous scatterplot of the *Feet* dataset [12]. (b) Rescaled using a selected by imgRV.

# 4.5 Effect of k in Anisotropic KDE

In addition, we present two examples to show the influence of the number of nearest neighbors (k) on the constructed anisotropic density field and the aspect ratios selected by imgRV in Figure 13 for the *Interleaved* dataset [9] and in Figure 14 for the *Statlog Image Segmentation* dataset [4].

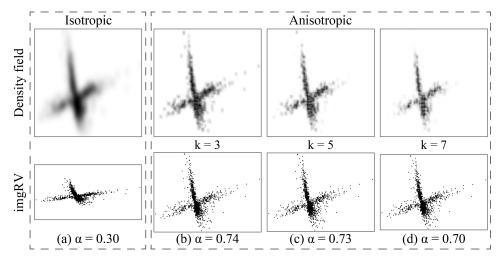


Fig. 13: Influence of the number of nearest neighbors (k) on the constructed anisotropic density field and the aspect ratios selected by imgRV on the *Interleaved* dataset [9]. (a) Density field constructed by isotropic KDE and its resulting plot; (b, c, d) density fields constructed by anisotropic KDE with different k and the resulting plots.

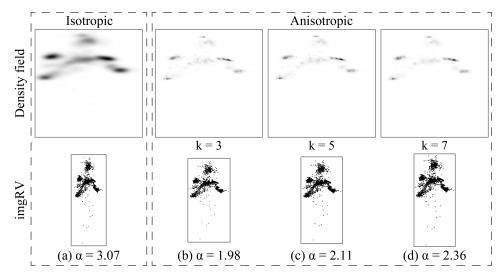
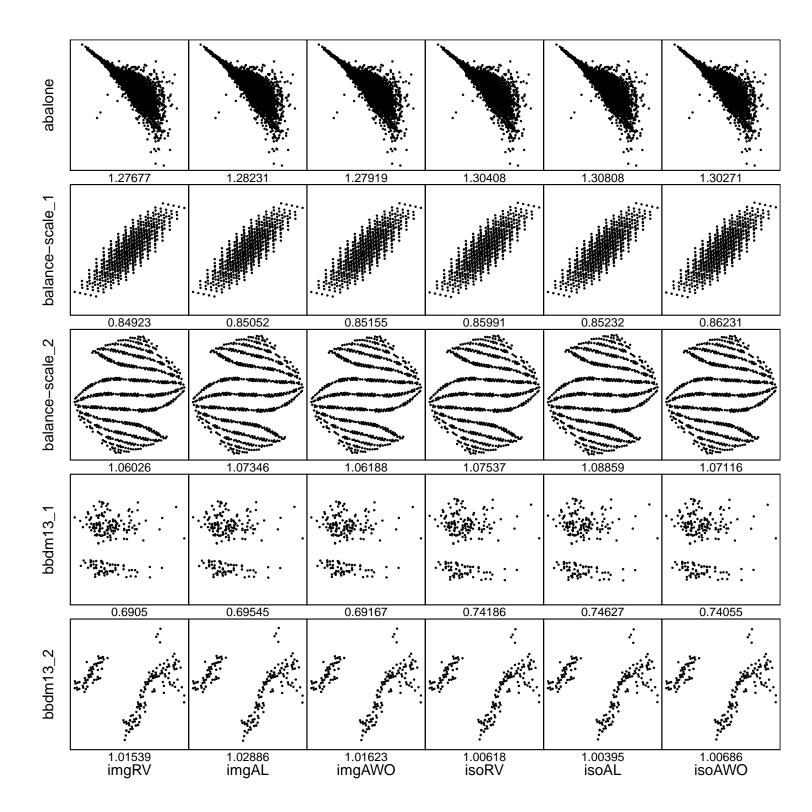
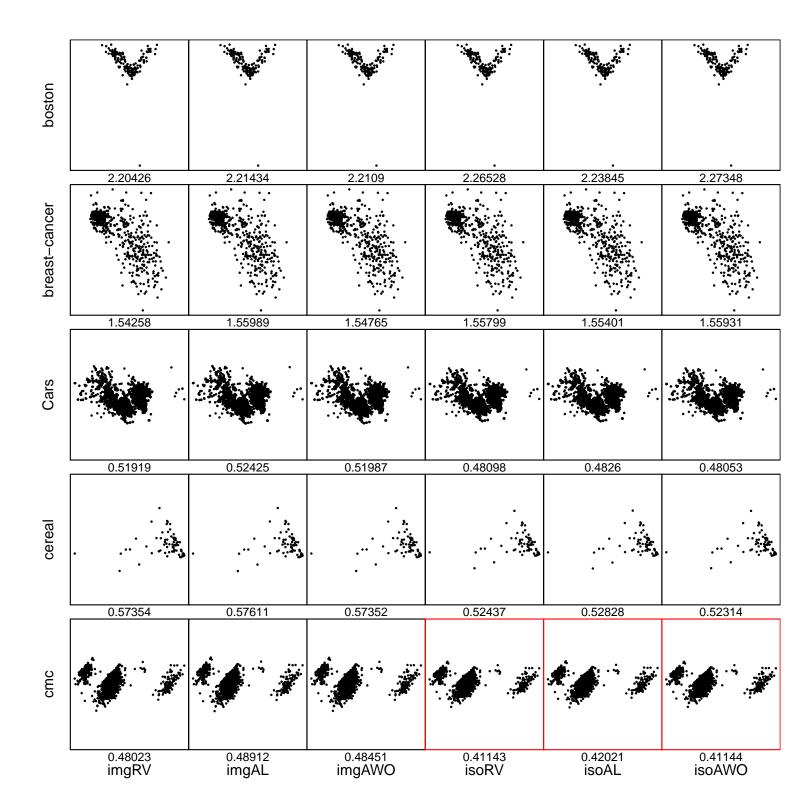


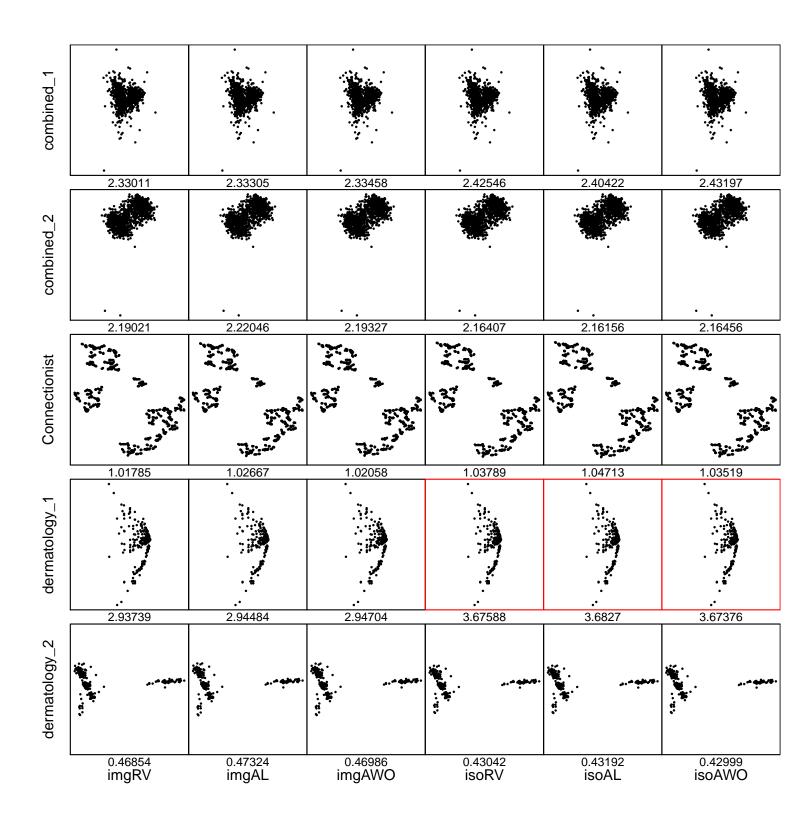
Fig. 14: Influence of the number of nearest neighbors (*k*) on the constructed anisotropic density field and the aspect ratios selected by imgRV on the *Statlog Image Segmentation* dataset [4]. (a) Density field constructed by isotropic KDE and its resulting plot; (b, c, d) density fields constructed by anisotropic KDE with different *k* and the resulting plots.

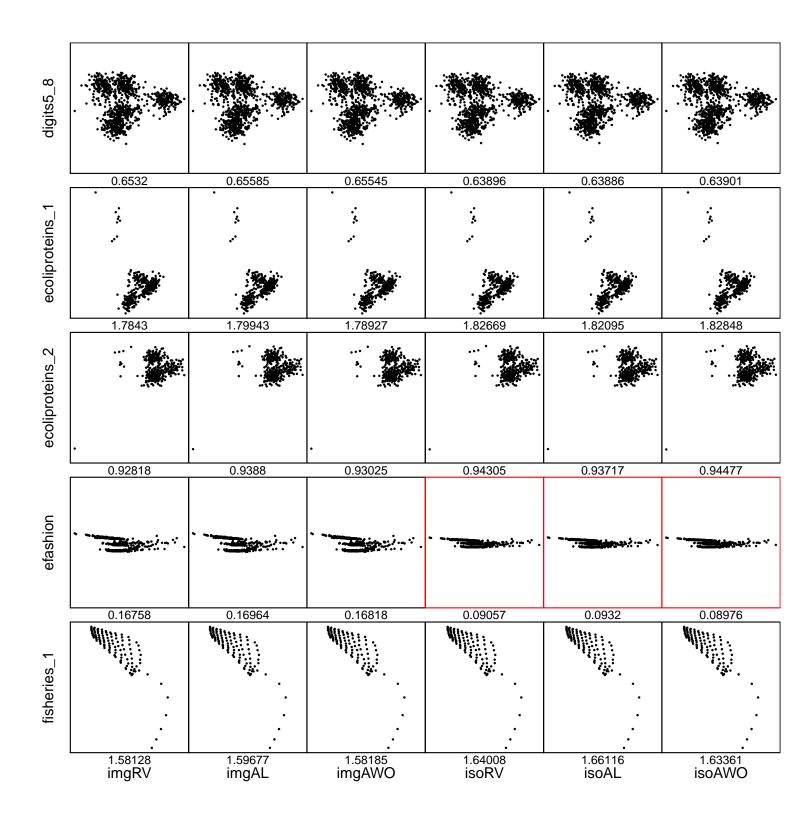
# APPENDIX A: COMPREHENSIVE COMPARISON OF SIX METHODS WITH 100 ISOVALUES

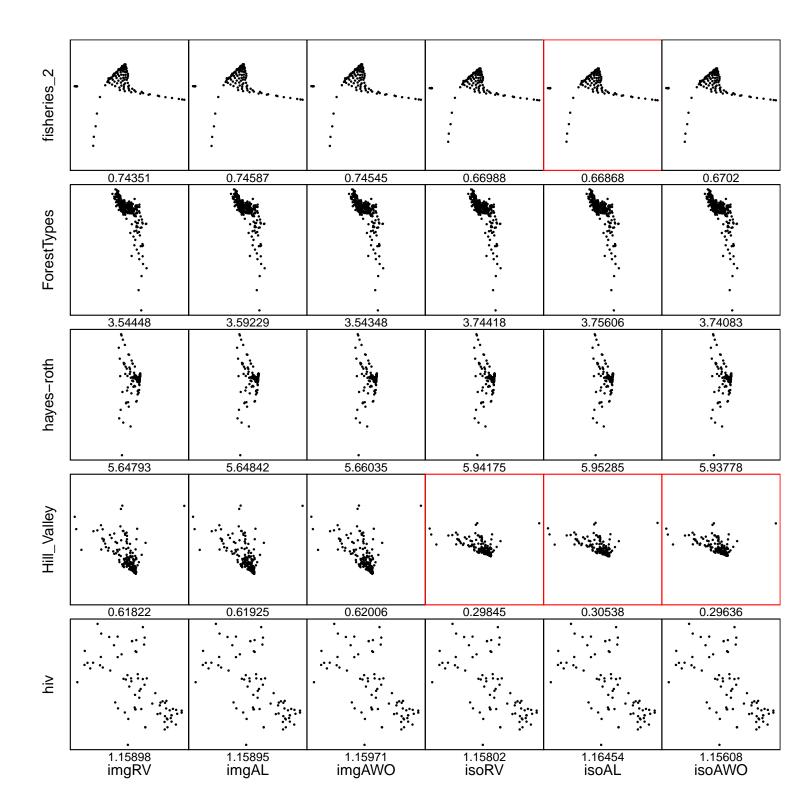
In this appendix, we present the detailed results with all 100 datasets from [4, 9] for comprehensive comparison of the six methods described in Section 2. Note that the number of isovalues m = 100. Moreover, the results whose deviations with isoRV (with m = 1000) are larger than 10% are plotted with a red bounding box.

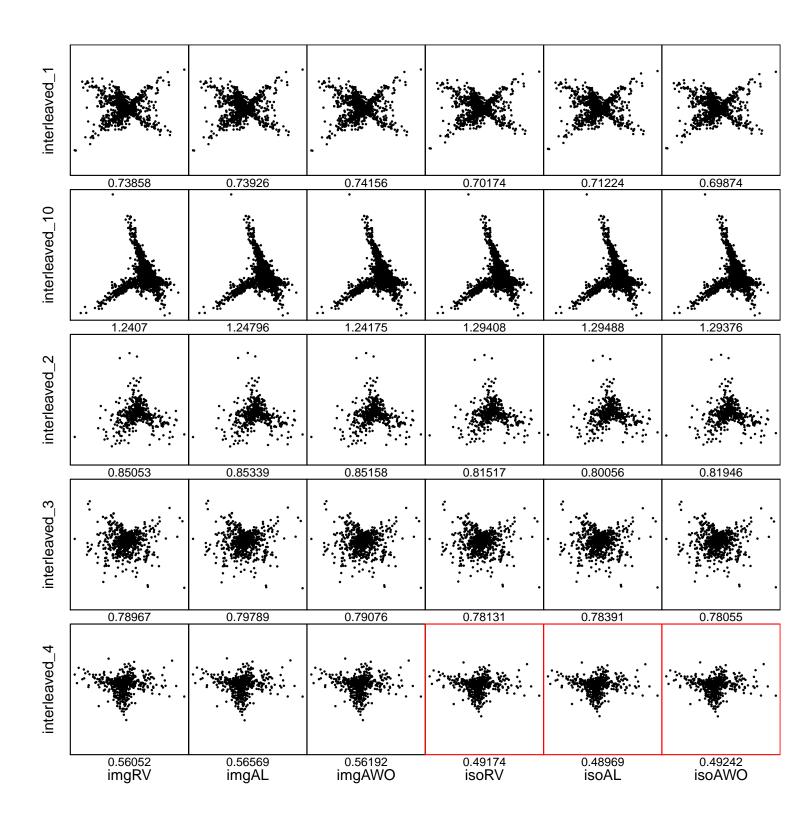


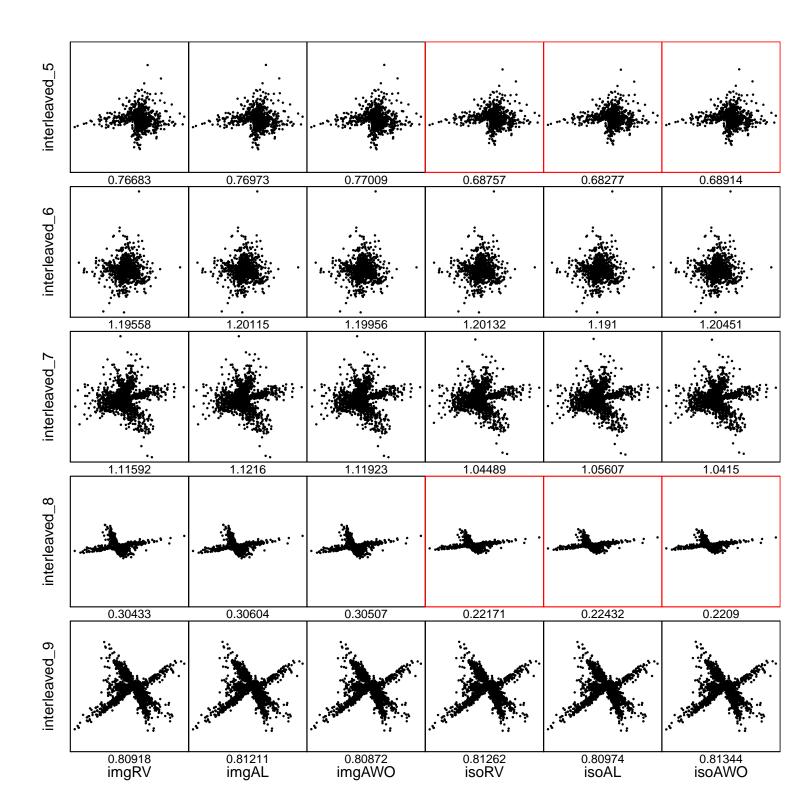


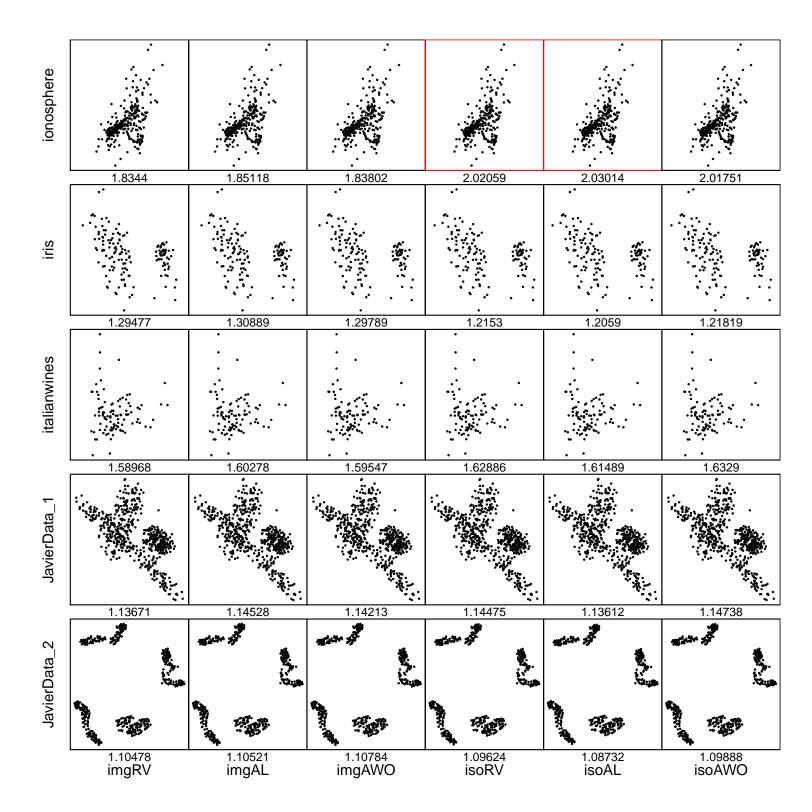


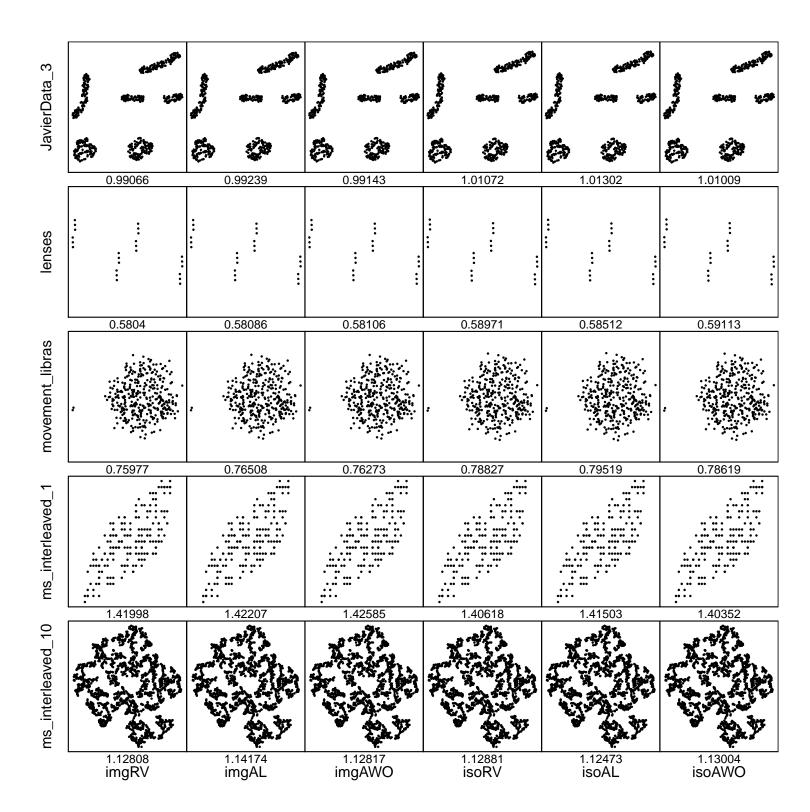


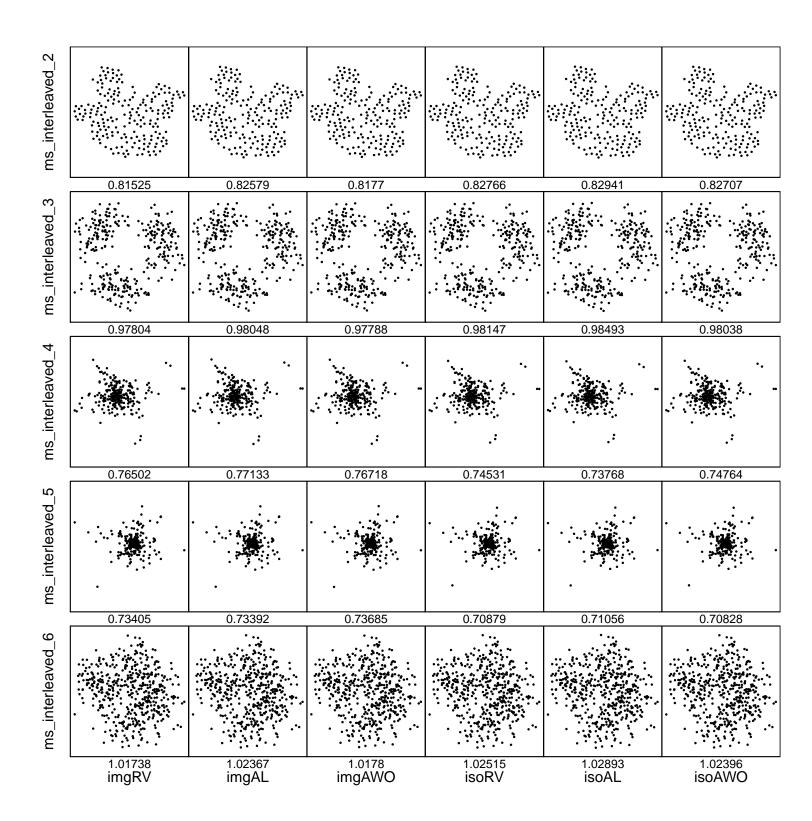


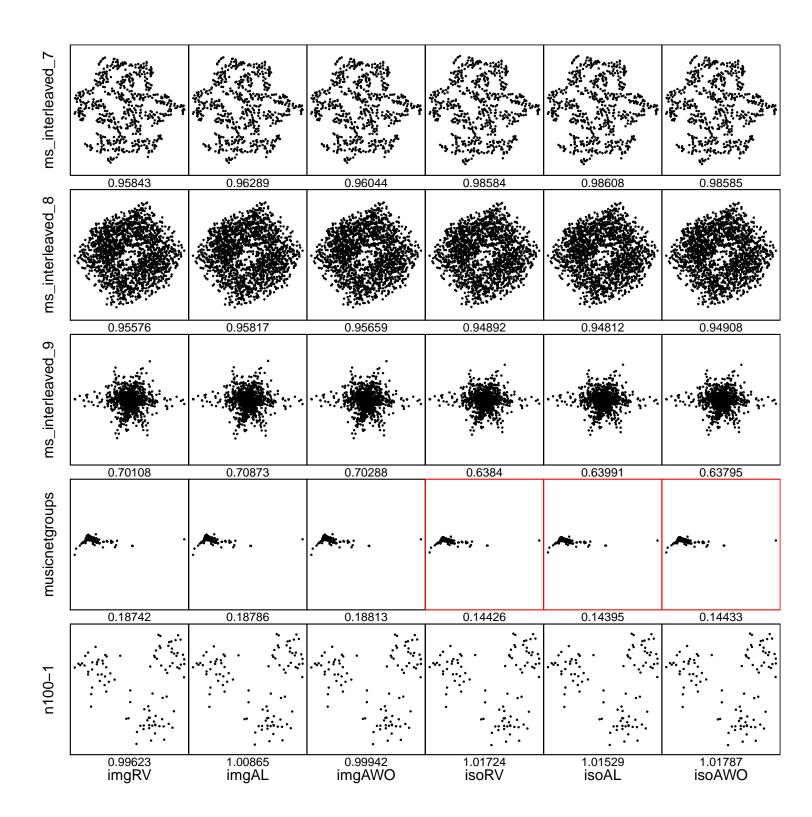


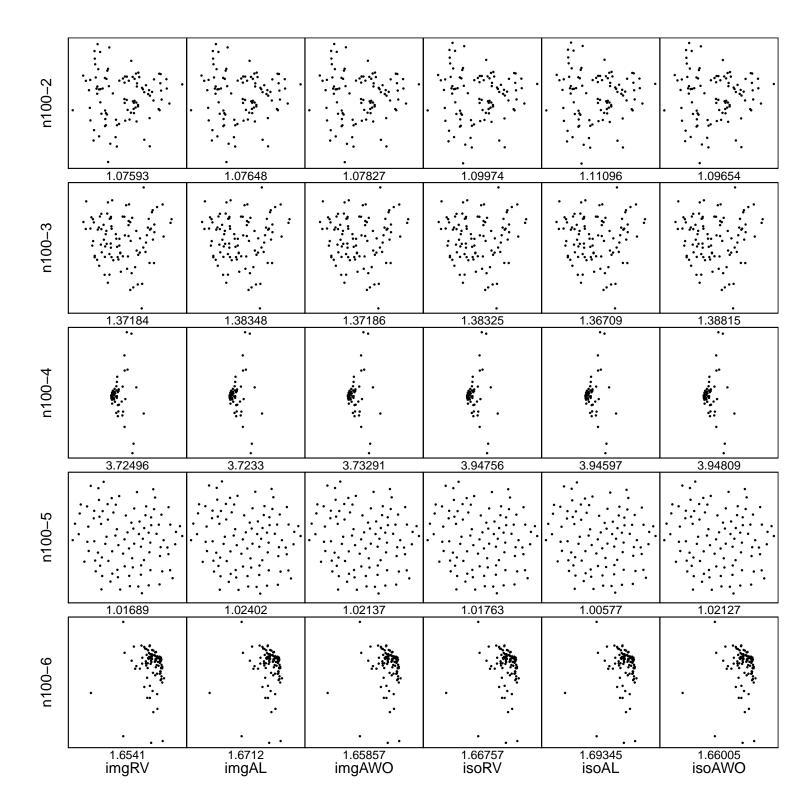


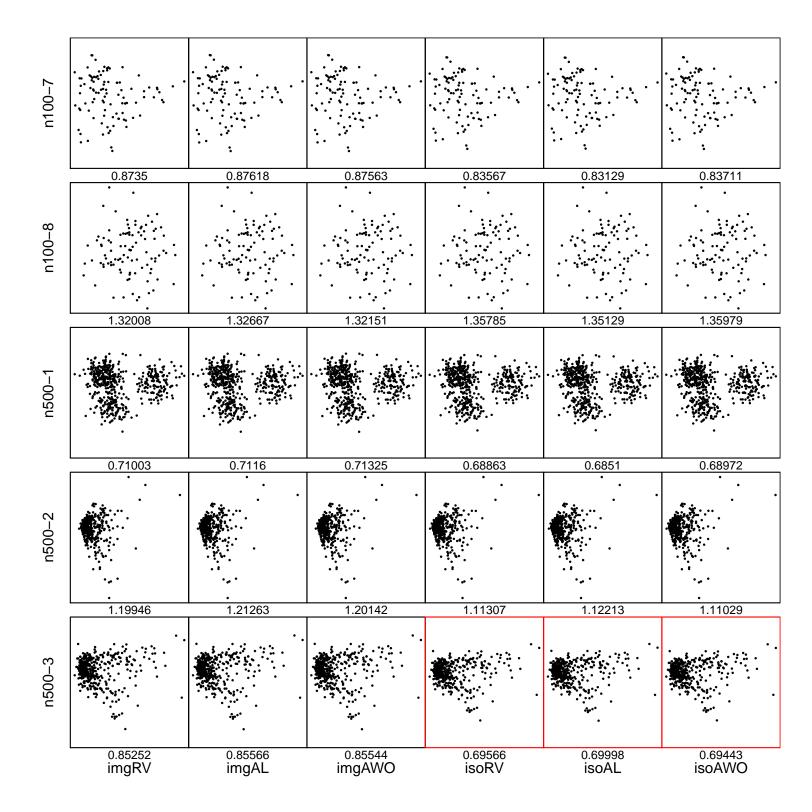


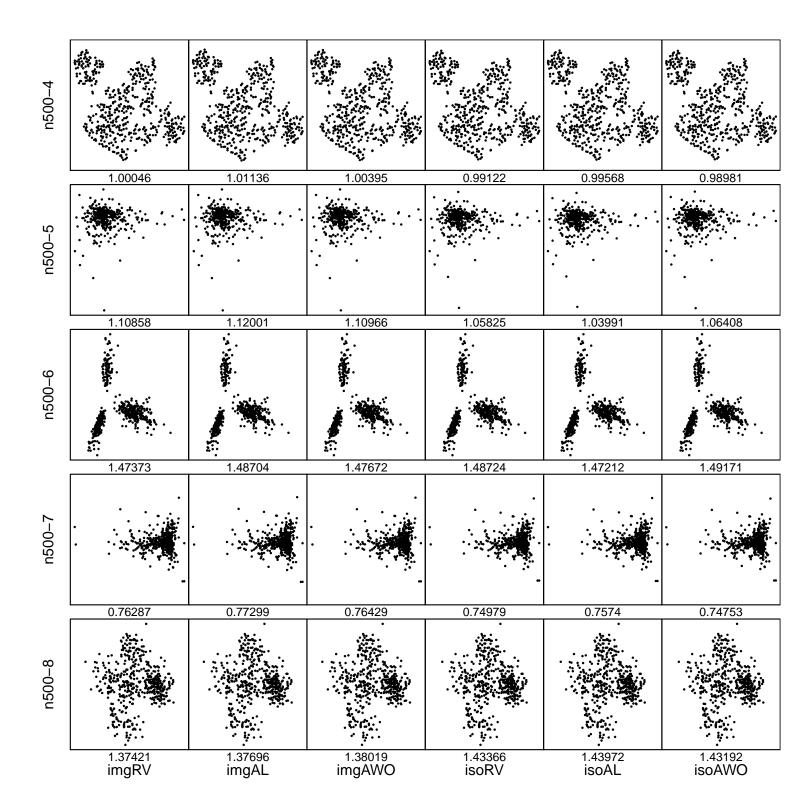


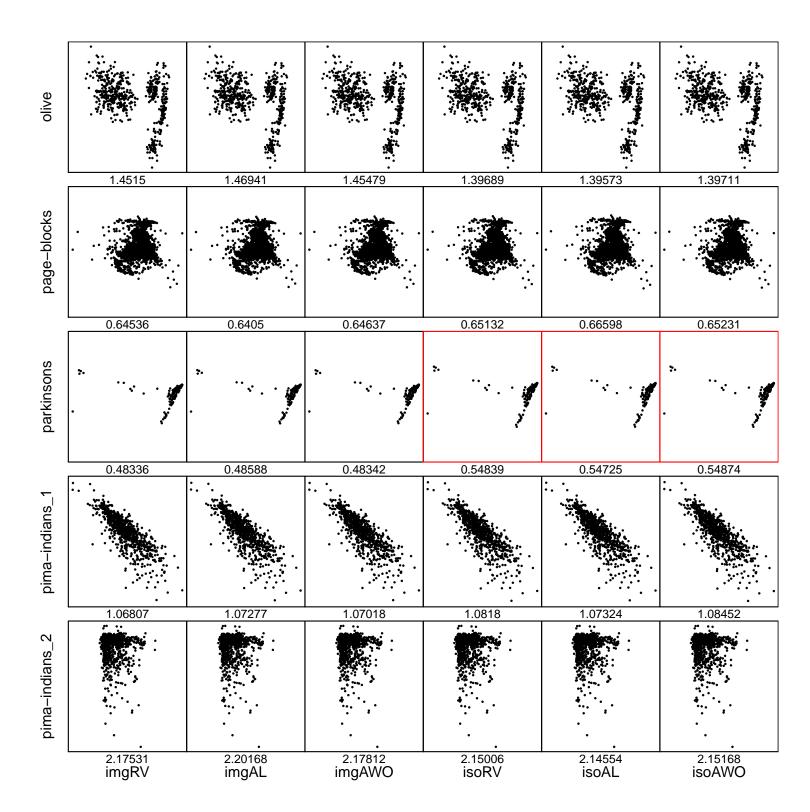


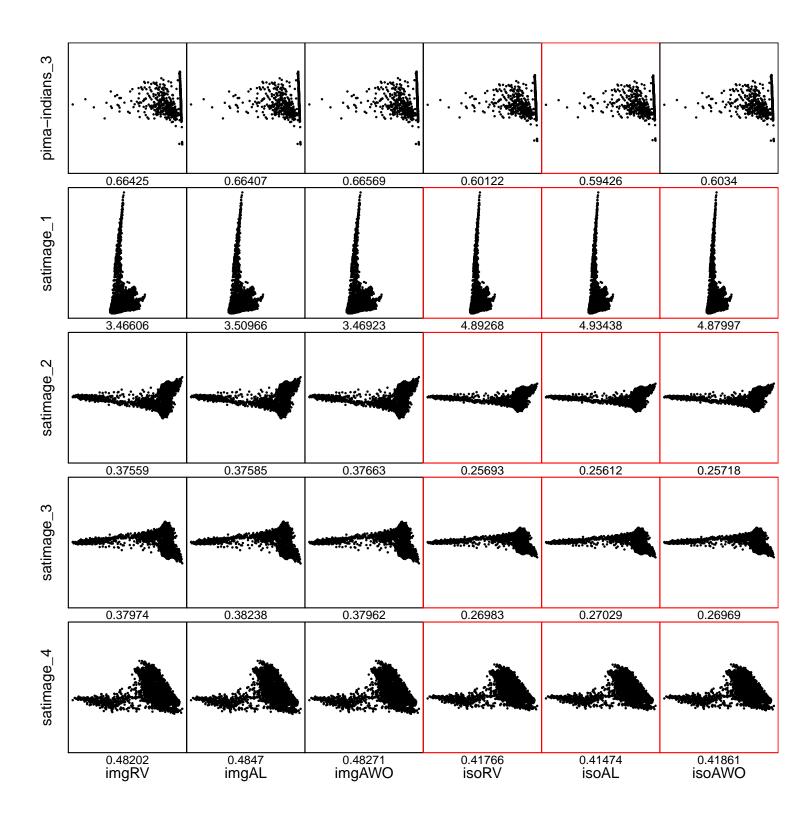


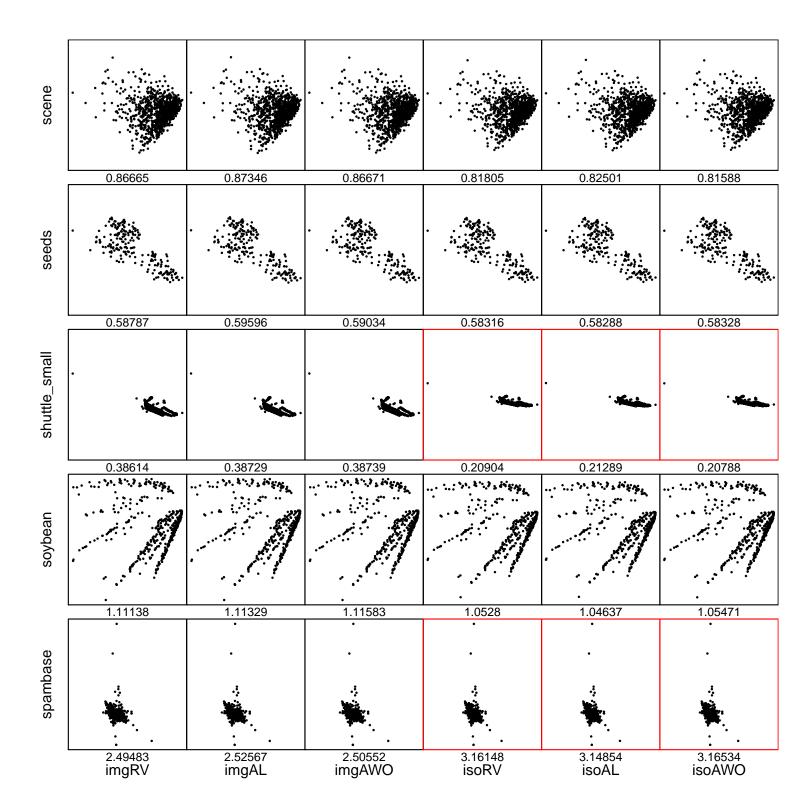


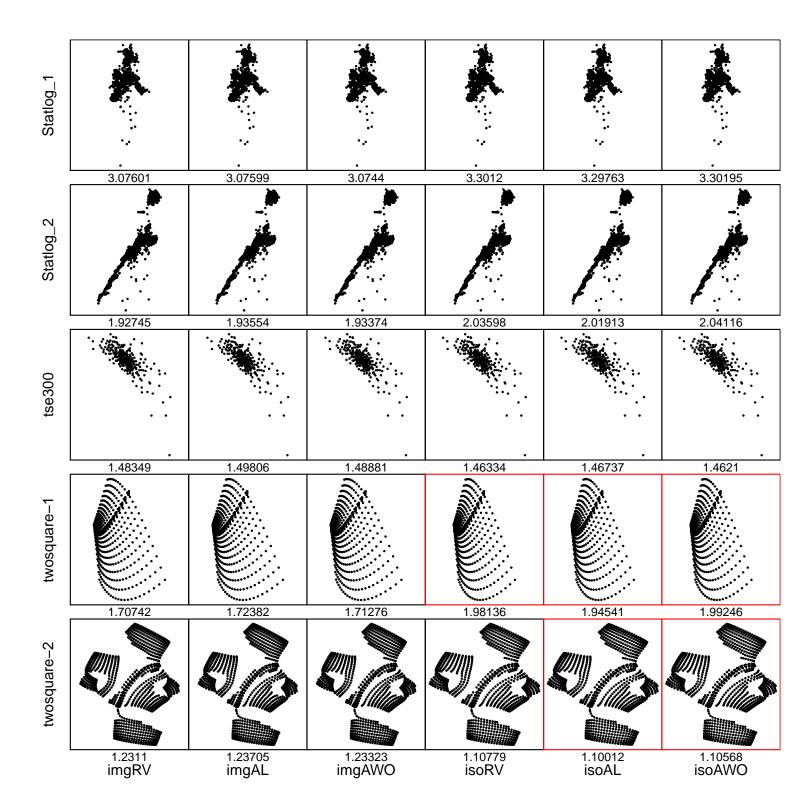


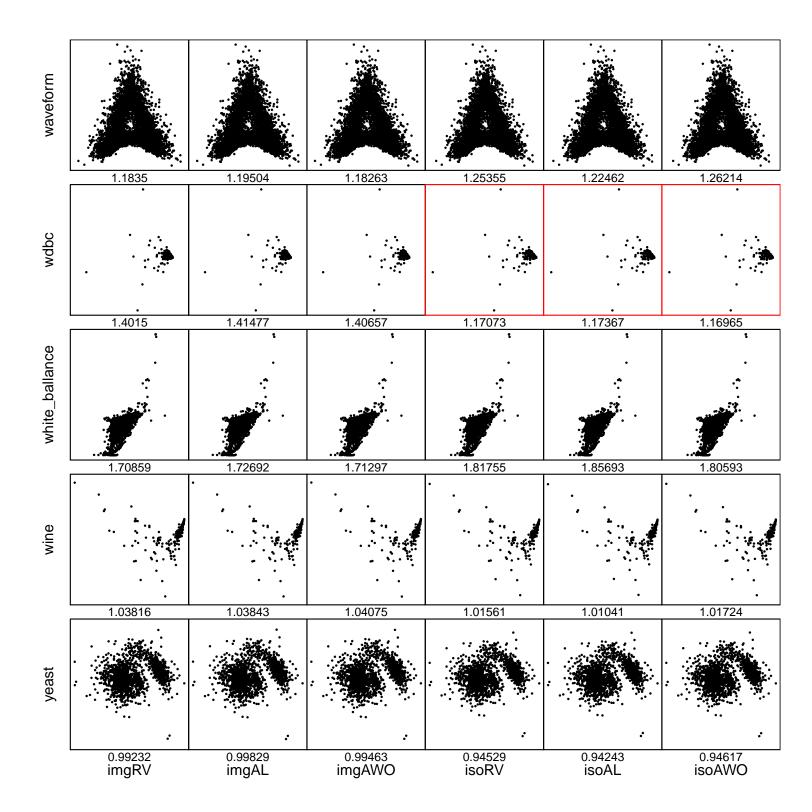






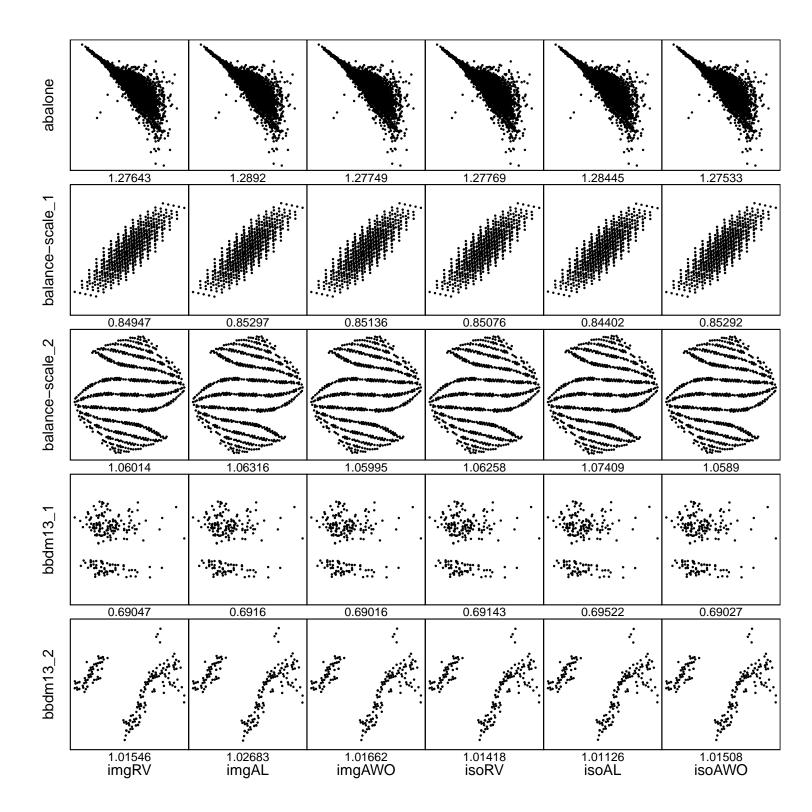


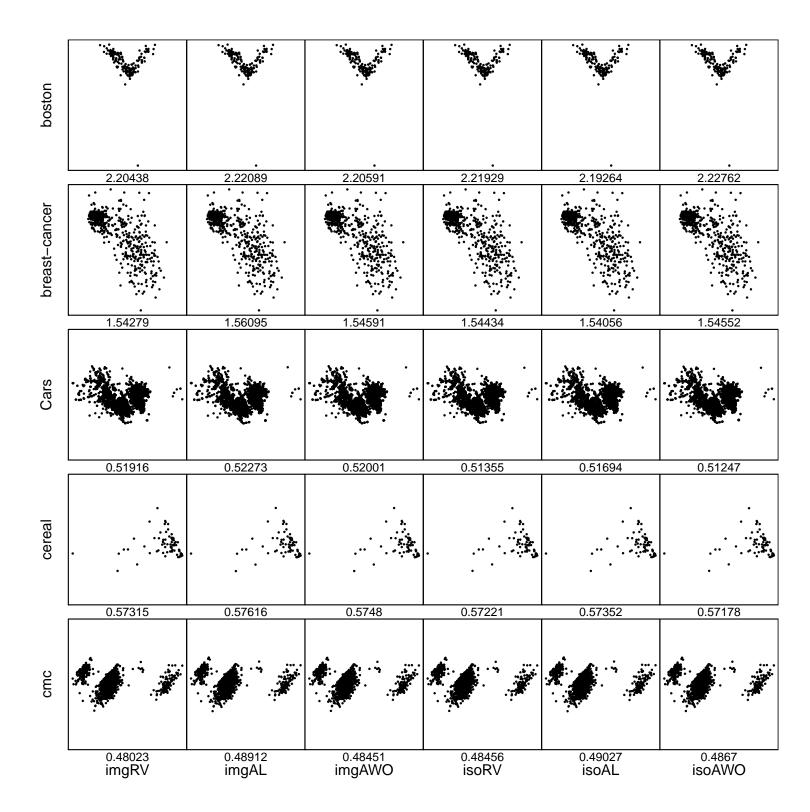


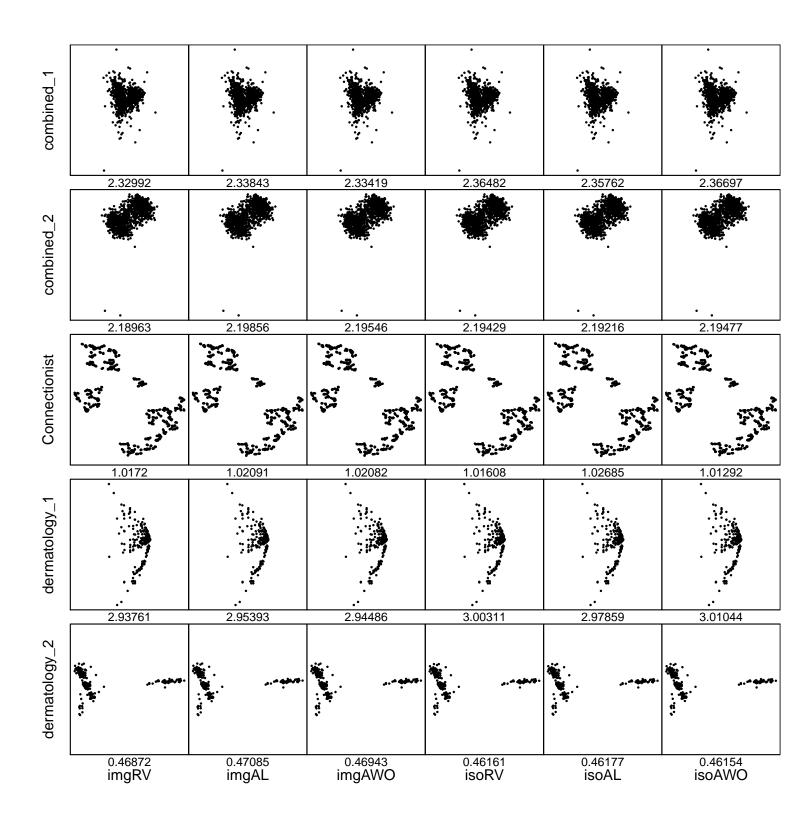


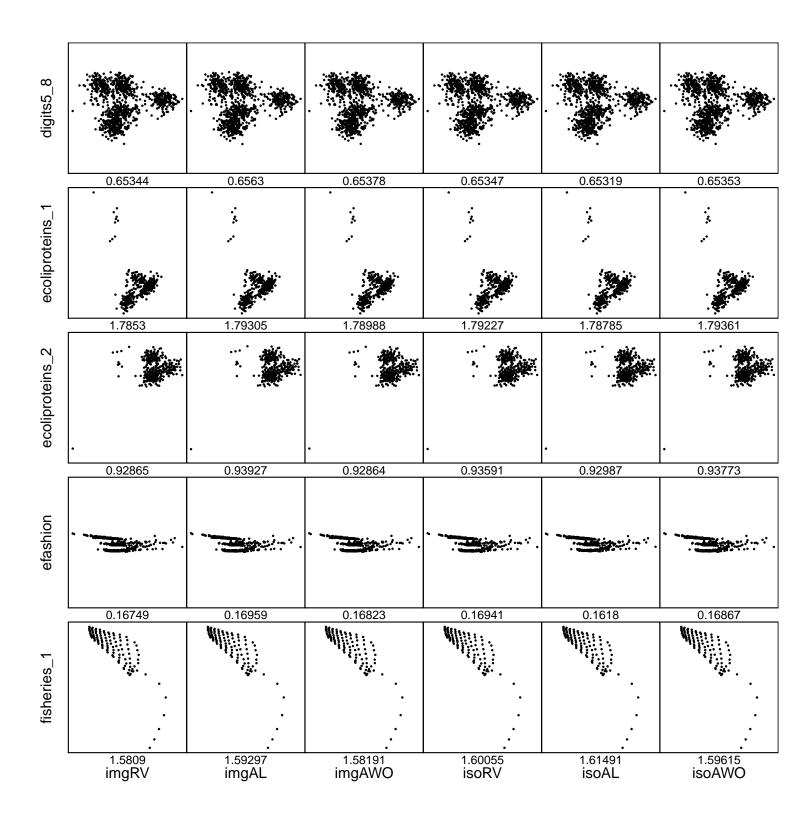
# APPENDIX B: COMPREHENSIVE COMPARISON OF SIX METHODS WITH 500 ISOVALUES

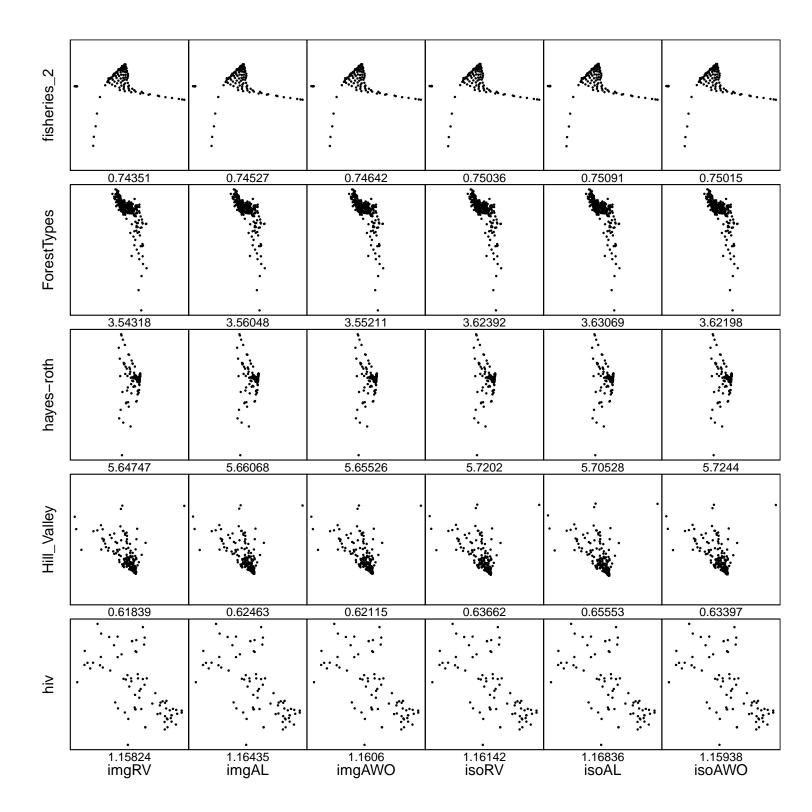
In this appendix, we present the detailed results with all 100 datasets from [4, 9] for comprehensive comparison of the six methods described in Section 2. Note that the number of isovalues m = 500. Moreover, there are no results whose deviations with isoRV (with m = 1000) are larger than 10%.

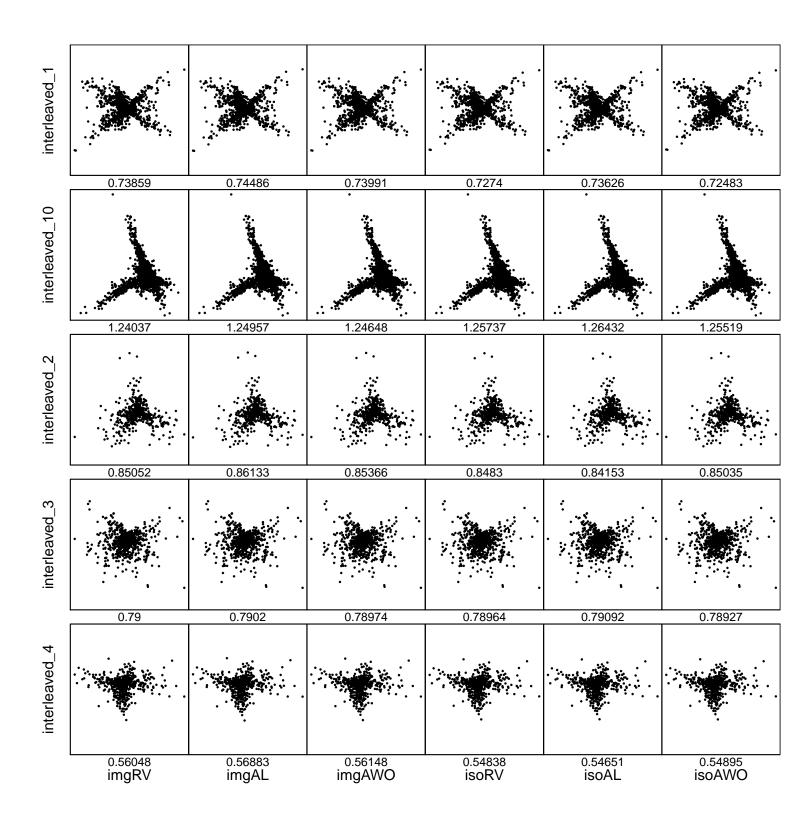


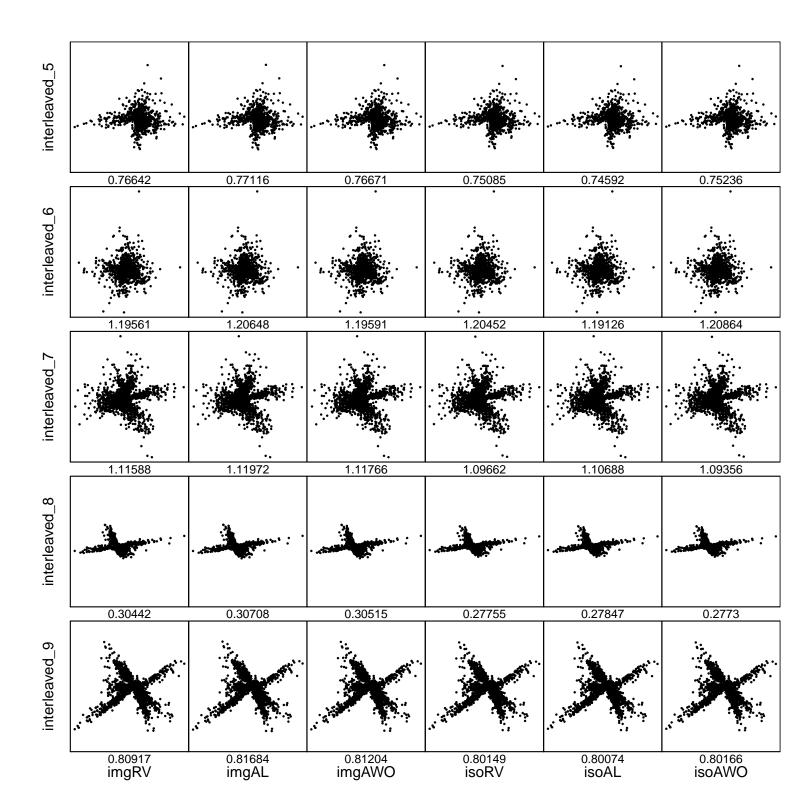


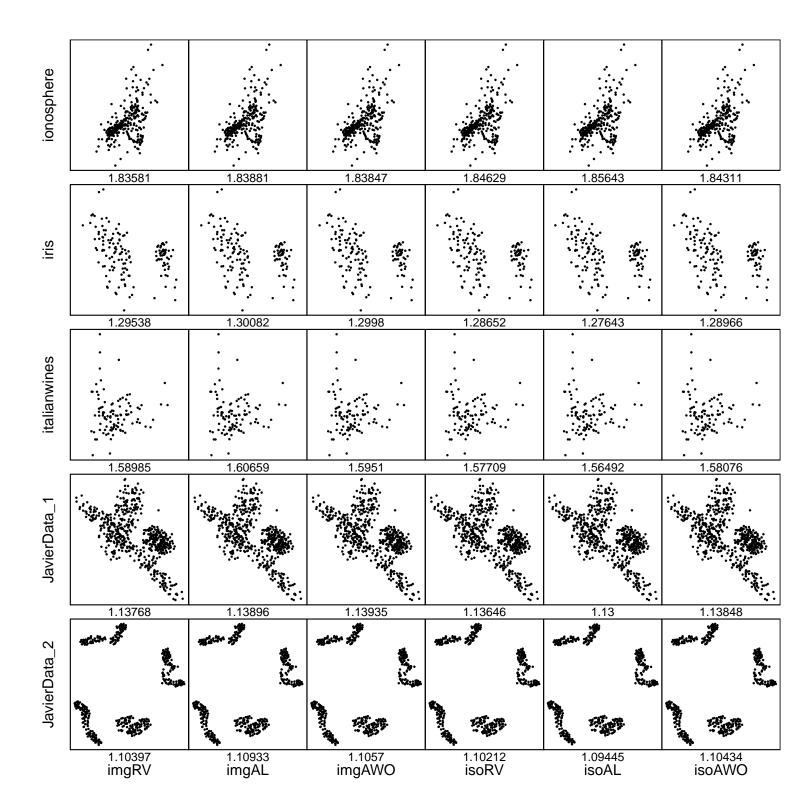


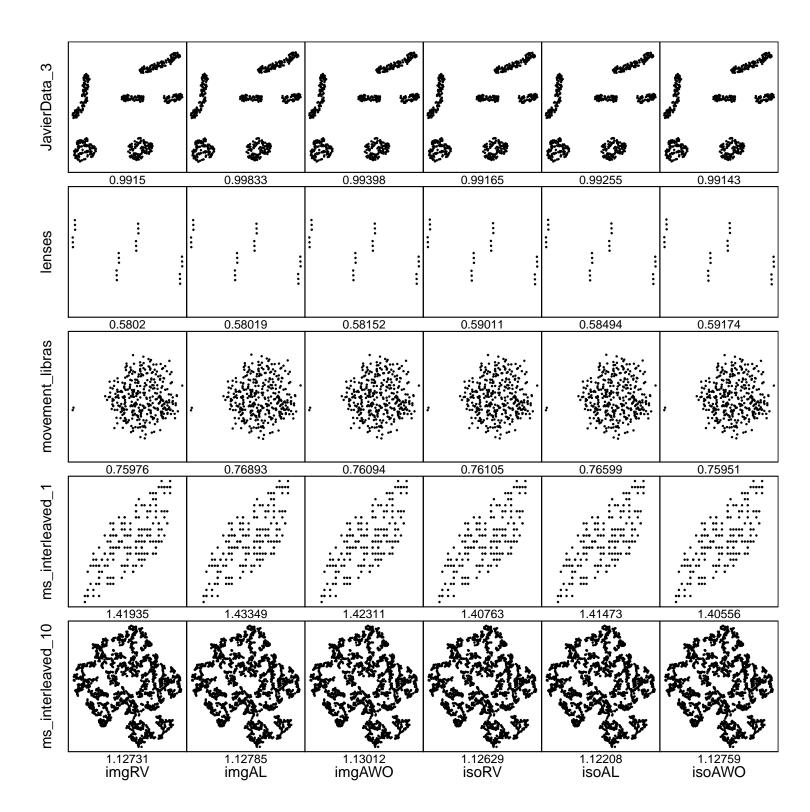


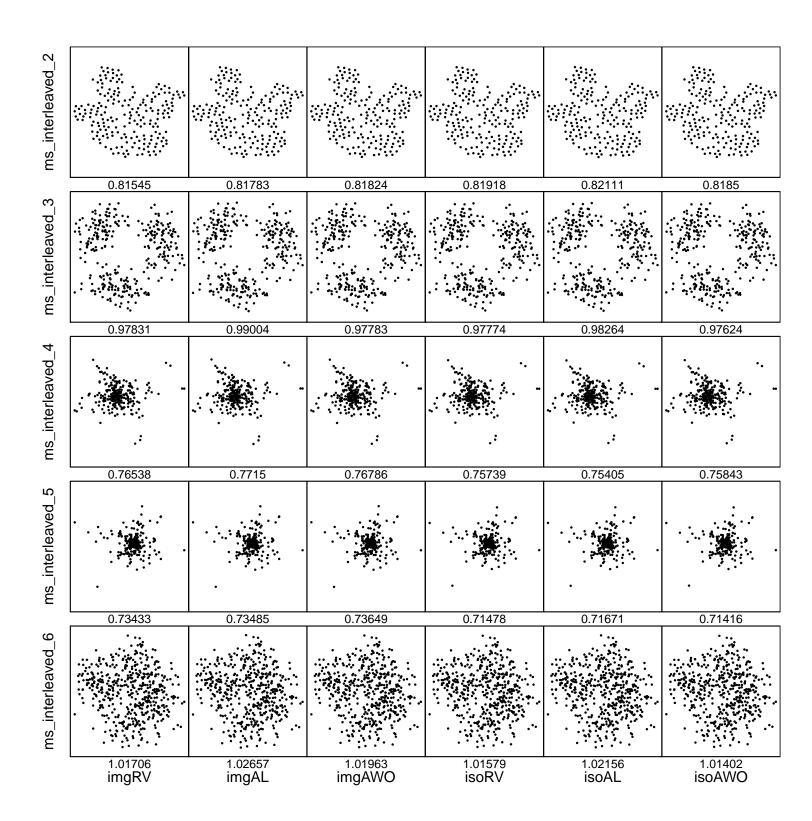


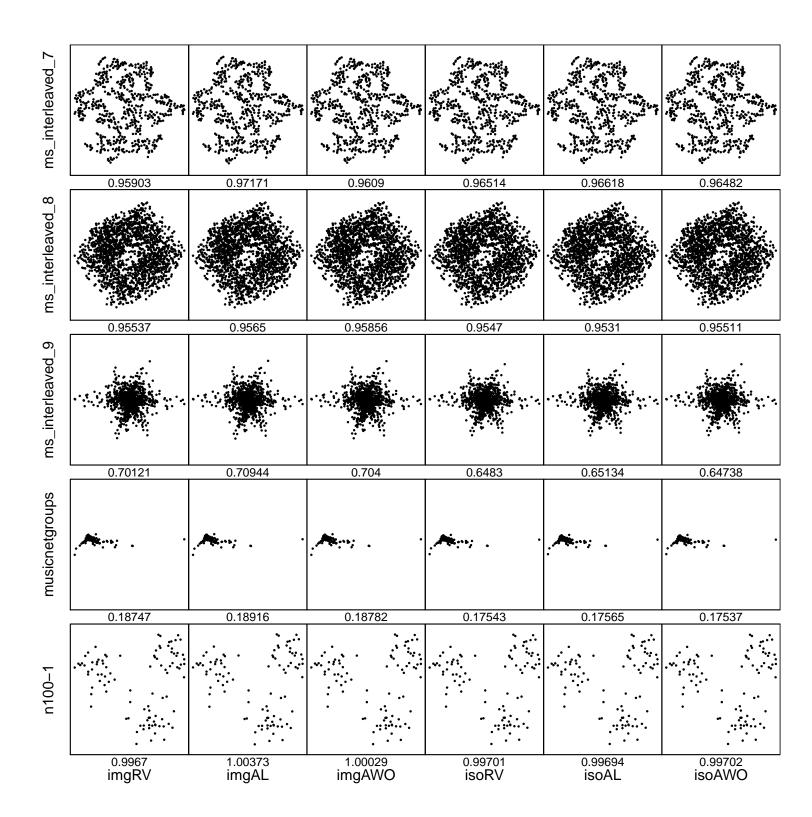


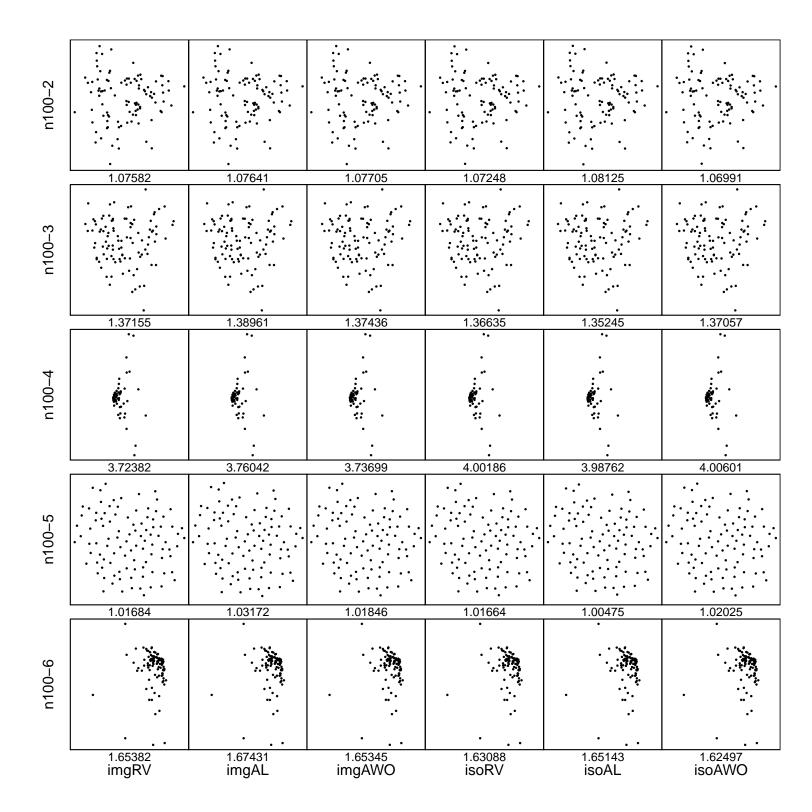


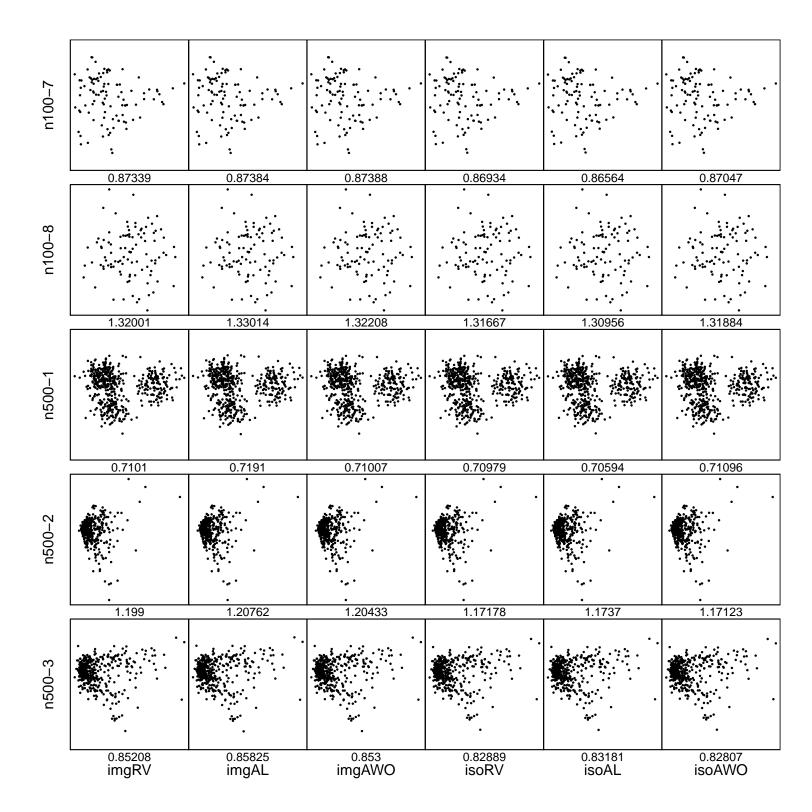


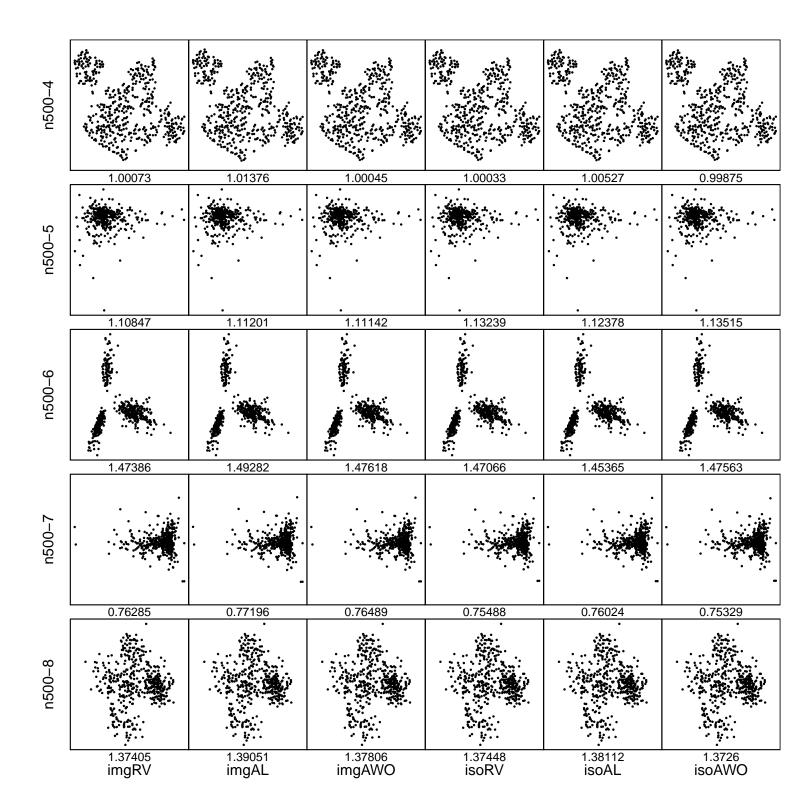


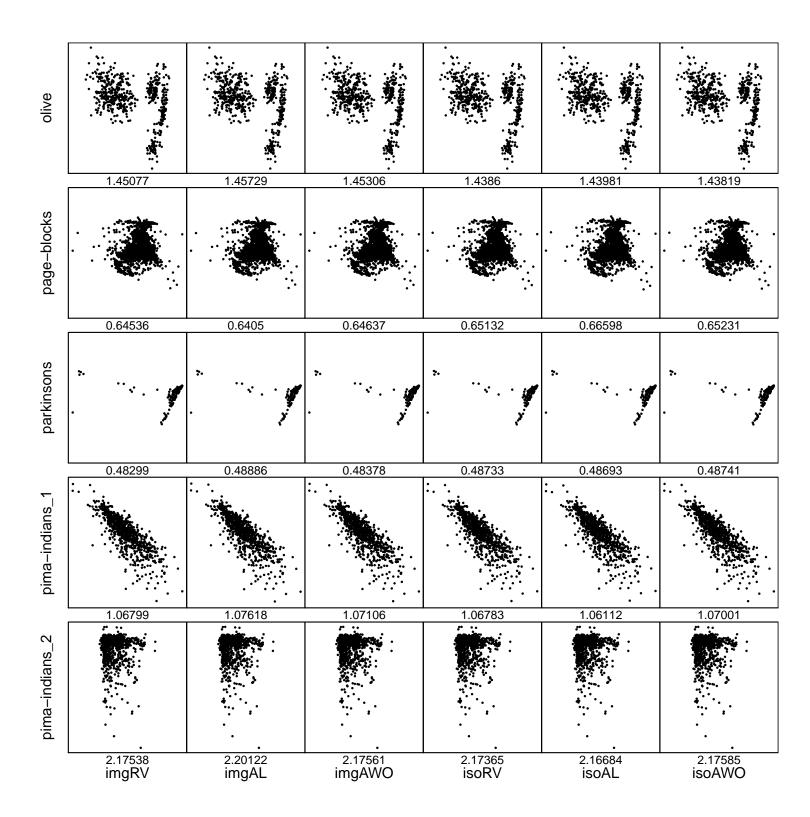


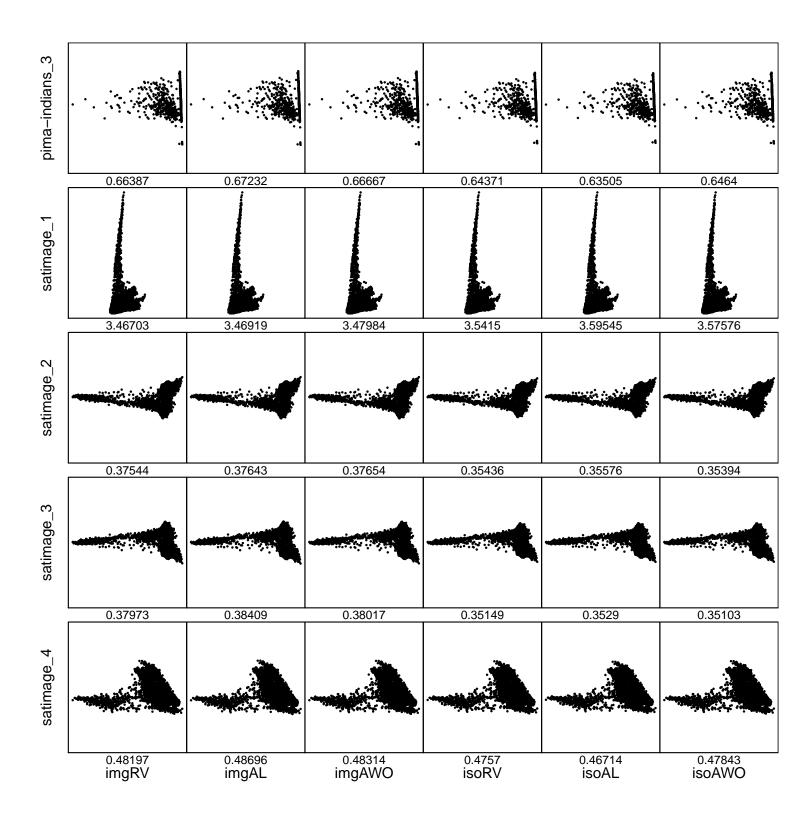


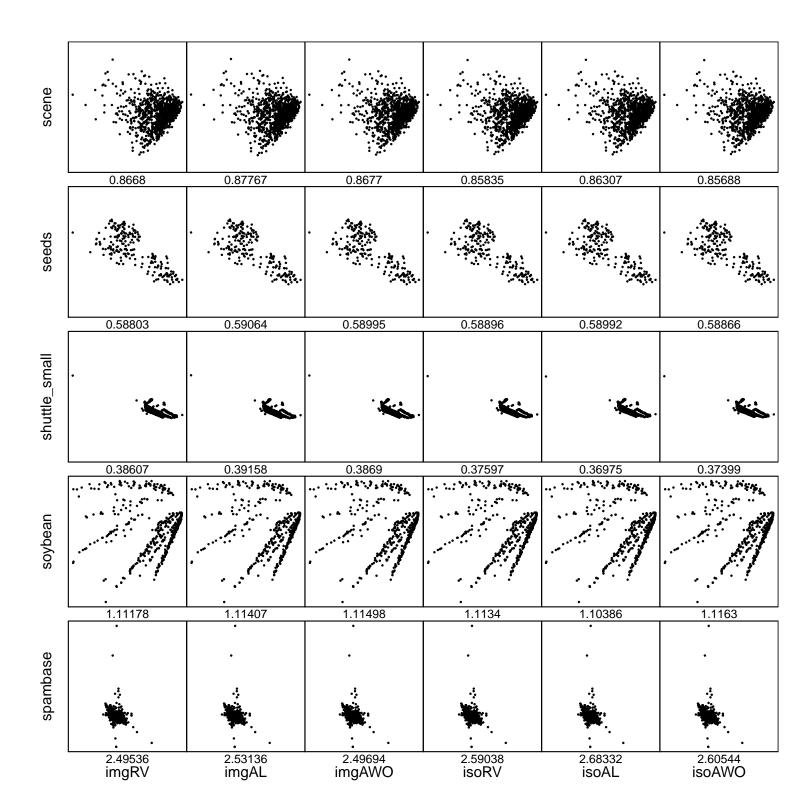


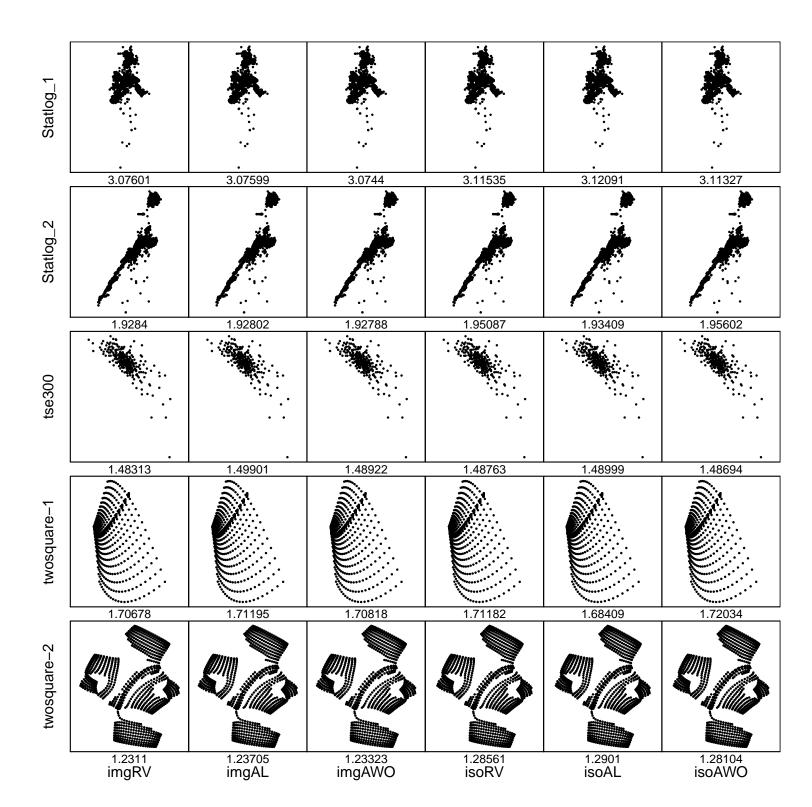


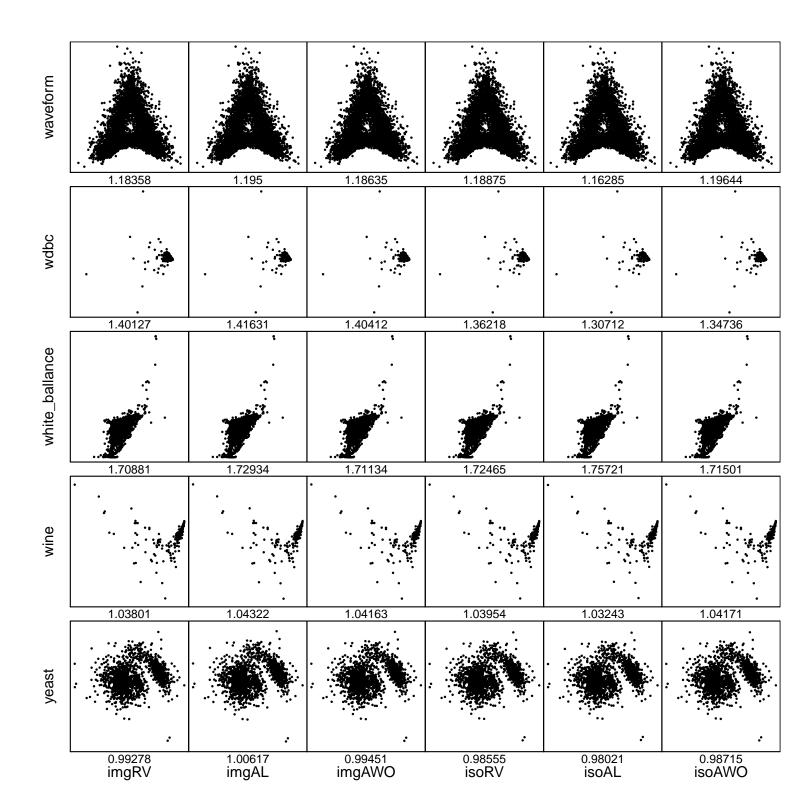












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